From bees and flowers to international trade networks (and other economical systems)

A complex network approach to economical systems

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Plan of the talk

- Introduction: what are we (physicists) doing here?
- What is a complex system? And why do we care about them?
- Complex Networks: a useful tool to study complexity
- Mutualistic ecosystems. The notion of nestedness.
- Applications to economical systems: import-export networks, industry location networks,
- Perspectives/discussion
Why?

- We are interested not only in the objects but in *the processes*. Theories of Phase Transitions and Critical Phenomena. Notions of scaling, universality, renormalization.

- Awareness of the pertinence of the scale at which the phenomena are described.

  *YES, we are aware that humans are different from electrons...*

- Notion of “model”: same word naming different concepts in physics and in other disciplines (eco, bio)
What is a complex system?

Lack of a unique definition, but a list of properties characterizing them:

- Number of elements: many, in general
- Type of interactions: non linearity, high correlations
- Emergent behaviour: self organization
- No hint about this organization by studying the properties of the components (the whole ≠ sum of the parts)
- Multiscale phenomena, interaction between different scales.
- Feedback loops

no leader
no plan
no master
A complex economical system
Easier to say what is not...

Complex ≠ Complicated!
Complex Networks

Abstract mathematical representation to account for very different systems in very different domains: nodes (vertices) - elements of the system, links (edges) - interactions between the elements.

Airport connections

Social links

Adjacency matrix

\[ A_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ connected} \\ 0 & \text{otherwise} \end{cases} \]

Different types:
- symmetric
- weighted
- directed

WWW network

Trophic network
Mutualistic ecosystems

**Bipartite networks** (adjacency matrices): two different kinds of vertices (animal and plant species); interactions only allowed between vertices of different kind

\[ K_{p,a} \in \{0,1\} \]

\[ D^P(p) = \sum_{a=1}^{n} K_{p,a} \]
For a perfectly nested system (ordered): the set of plant (animal) species associated to a given animal (plant) is included in the set of plant (animals) species associated to the previous ones.

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The ecosystem is composed of “specialists and generalists” species

First measurement of nestedness:
Atmar-Patterson « temperature » ($T_{AP}$)
Measuring nestedness

Atmar and Patterson « temperature » (APT) : A geometrical characterization of the adjacency matrix that measures the departure of the real system from the IPN. APT \( \sim 0 \) indicates a nested system while APT \( \sim 100 \) correspond to a disordered system.

**Problem:** \( T_{\text{AP}} \) very sensitive to the n/m ratio, the density of contacts...

An operational nesting coefficient based on robustness.


The system can suffer from **random** or **targeted** attacks

For an ordered system, two extreme targeted strategies

- Starting from the smallest \( k \) \((- \rightarrow +)\)
- Starting from the largest \( k \) \((+ \rightarrow -)\)

The resistance of the system to the attack (robustness of the network), depends on its organisation it can be used to measure its degree of order
Attack tolerance curve (ATC): It gives the fraction of survival species as a function of the fraction of the attacked counterpart: $f_s(f_k)$

Robustness coefficient

$$R^{++} = \int_0^1 f_s(f_k) df_k$$

for a perfectly nested system

\[ k_1 < k_2 \]

\[ m \]

\[ p \]

\[ n \]

Attack strategy

Open symbols: $(- \rightarrow +)$

Full symbols: $(+ \rightarrow -)$

\[ R^{\rightarrow+} = 1 \] and \[ R^{+\rightarrow} = \phi \]
The nesting coefficient

\[ N_{c(r)} = \frac{R_{c(r)}^+ - R_{c(r)}^-}{1 - \phi} \]

N=1 for perfectly nested system
N<<1 for random system

Some values for real systems

<table>
<thead>
<tr>
<th>Adjacency matrix</th>
<th>( \phi )</th>
<th>( T_{AP}^* )</th>
<th>( N_c )</th>
<th>( N_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Clements</strong></td>
<td>3.4%</td>
<td>0.16</td>
<td>0.61</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Robertson</strong></td>
<td>2.3%</td>
<td>0.06</td>
<td>0.65</td>
<td>0.58</td>
</tr>
<tr>
<td><strong>Kato</strong></td>
<td>1.9%</td>
<td>0.11</td>
<td>0.75</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Why nestedness? The SNM Algorithm


Objective:

Trial of different dynamical rules and their corresponding equilibrium states

inputs: n,m, density of contacts (ϕ), random initial adjacency matrix.

Basic requirements:

- Local rules.
- Description of the partially ordered situations found in nature.
- Asymptotical approach to the perfectly nested state given by the corresponding IPN.

Basic Assumption:

The number of interacting species is fixed → No extinction is allowed.
Contact Preference Rule (CPR)

The allocation of contacts will be performed according to the CPR to be tested:

- **SNM-I**: a swap is accepted if the new counterpart’s degree is higher than the old one.
- **SNM-II**: a swap is accepted if the new counterpart’s degree is lower than the old one.

**General remarks**

- **Biological plausibility**: SNM-I promotes the contact with generalists (enhanced competition vs. increased efficacity and longevity).
- **SNM-I and preferential attachment**: No growing process here, and completely local rule.
1-step of the SNM dynamics

1. Choose alternatively a row (column) at random.

2. In that row (column) locate two columns (rows): one with 0 and the other with 1.

3. Perform a swap between 0 and 1, provided the CPR is verified.

4. Choose a row (column) at random and continue iterating.

**Stopping criteria**

- Same $T_{AP}$ as the corresponding real system.
- Agreement with the degree distribution of the real system.
SNM-I / Clements and Long system

SNM-initial random matrix

SNM-1000 steps

SNM-2175 steps (same T as real system)

Real system

SNM equilibrium state
Comparison between real and simulated system:

Degree distributions for plants and animals are compared between the real system (Robertson) and the SNM model at the same temperature (T). The graphs show the cumulative degree distribution in the bipartite lattice, with axes representing degrees and the cumulative distribution function.
Transition from the initial random state to the ordered nested state

ATC

Robustness, nesting coefficient

in all cases, averages over 200 initial random networks
Asymmetric SNM: model for mutualistic social systems

R = \frac{P_r}{P_c}, \quad P_r, P_c \text{ updating frequency of rows and columns respectively}

5 \times 10^6 \text{ iterations, all converged}
The asymmetric SNM: ecological vs. social systems

Inset: degree distribution
The amount of knowledge embedded in a society does not solely depend on the amount of knowledge each individual holds. It mainly depends on the interactions among those individuals: their ability to combine this knowledge to make use of it — complexity.

The complexity of an economy is related to the multiplicity of useful knowledge embedded in the society. This is reflected in the society’s productive output.
Bipartite Network’s approach

important: not only how many but which one?

diversification of a country

ubiquity of a product

Bipartite matrix: $M_{cp} \in \{0,1\}$
Tools for analysing bipartite networks: The projected networks

\[ W_{c,c'}^C = \sum_p K_{c,p} K_{p,c'}^T (1 - \delta_{c,c'}) \]

\[ W_{p,p'}^P = \sum_c K_{a,p}^T K_{c,p'} (1 - \delta_{p,p'}) \]
The degree of each country \((a)\) or product \((\alpha)\) in the bipartite matrix:

\[
k_{a,0} = \sum_{\alpha} M_{a\alpha} \quad \kappa_{\alpha,0} = \sum_{a} M_{a\alpha}
\]

One can build the following \(N\)-iteration process:

\[
k_{a,N} = \frac{1}{k_{a,0}} \sum_{\alpha} M_{a\alpha} \kappa_{\alpha,N-1} \quad \kappa_{\alpha,N} = \frac{1}{\kappa_{\alpha,0}} \sum_{a} M_{a\alpha} k_{a,N-1}
\]

The first terms are easy to interpret:

\[
k_{a,1} = \frac{1}{k_{a,0}} \sum_{\alpha} M_{a\alpha} \kappa_{\alpha,0} \quad \kappa_{\alpha,1} = \frac{1}{\kappa_{\alpha,0}} \sum_{a} M_{a\alpha} k_{a,0}
\]

average ubiquity of products made by country \(a\)

average diversification of countries that produce product \(\alpha\)
Iterating (2) for $N \to \infty$, one gets an eigenvalue problem, using the second largest eigenvalue (the largest is 1) one can build the ECI, which is used to rank the countries.
Comparison with other indices

- Relationship between Years of Schooling and the Economic Complexity Index (ECI) for the year 2000.

- Relationship between Cognitive Ability and the Economic Complexity Index (ECI) for the year 2000.
Communities in product space

As two co-exported products carry similar knowledge: a hint to infer new products to come.
Nestedness in economical networks

Bustos, Gomez, Hausmann, Hidalgo PlosOne 7 e49393 (2012)

Study of the worldwide country-product network and national municipality-product network in Chile

grey: state in 1993

international local (Chile)
Study of the robustness of the system, using ATC. A way to compare robustness of different countries/regions?

Usefulness of magnitudes issued from ecology: ecological diversity

\[ S_{pp'} = \frac{W_{pp'}}{D_p + D_{p'}} \]

\( D_p \) : degree in the projected network

SNM to study stability of nestedness according to some CPR

Application to the study of the organisation of other kind of economical systems?
Complexity in social systems: from data to models

27-28 juin 2013

Université de Cergy-Pontoise - Site de Saint-Martin, 2 av. Adolphe-Chauvin à Pontoise

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- Irene Vodenska, Boston University, U.S.A.


Thank you for your attention!