Long Horizon Predictability:
An Asset Allocation Perspective

Abraham Lioui* and Patrice Poncet**

Presented at « Séminaire Interne du LABEX MEE-DII»

CNIT La Défense, February 26, 2012

*   EDHEC Business School, Nice, France. abraham.lioui@edhec.edu
** ESSEC Business School, Cergy, France. poncet@essec.fr

Outline of Presentation

- **1. Motivation**
- **2. Framework**
- **3. Estimation**
- **4. Portfolio allocations**
- **5. Correcting for overlapping observations**
- **6. Out-of-sample analysis**
- **7. Take away & Ongoing research**

1. MOTIVATION

- While the extent of Asset Return Predictability (ARP, e.g. Fama & Schwert (1987); Campbell (1987)) is somewhat debated, there exists huge literature documenting time variation in first (and second) moments of stock returns

- ARP issue is clearly relevant to market efficiency, asset allocation, and welfare

- A typical predictive regression is:

$$r_{t+1} = a + b\, z_t + \varepsilon_t$$

with typical predictors: Dividend yield, T-Bill rate, Term and/or Default Spread, Earnings/Price ratio, Realized Volatility (Bollerslev et al., …).

- Using monthly returns on the NYSE-NASDAQ-AMEX index, the dividend yield (*dy*) on the same index, and a default spread measure (*def*) one has:

$$r_{t+1} = -0.01 + \underset{(2.97)}{0.32}\, dy_t + \underset{(0.38)}{0.16}\, def_t + \varepsilon_{t+1} \qquad Adj.R^2 = \mathbf{0.01}$$

- Using 5-year returns, we have:

$$r_{t+60} = -0.15 + \underset{(5.92)}{14.88}\, dy_t - \underset{(1.08)}{7.01} def_t + \varepsilon_{t+60} \qquad Adj.R^2 = \mathbf{0.40}$$

[*t*-stats are Newey-West (autocorrelation & heteroskedasticity)]

- Is the multiplication of R^2 by around **40** economically meaningful?

►Beware: Predictive regressions are known to suffer from problems even *in sample*.

In particular: if innovations in z and ε are correlated, there is a bias in b. Stambaugh (1999), Amihud-Hurvich (2004), and Amihud-Hurvich-Wang (2008) provide corrections. These, however, cannot be applied here with OD, because of serial correlation in residuals of the OLS predictive regression(s).

- Short horizon ARP: weak but there; e.g. Ferson & Harvey (1999), Goyal & Welch (2005), Lewellen (2004), Menzly et al. (2004), Santos & Veronesi (2006), Lettau & Van Nieuwerburgh (2008), Ferreira & Santa-Clara (2011), Yu (2011), Pastor & Stambaugh (2011).

- Long horizon ARP is less consensual. "*Conventional academic wisdom*" (e.g. Fama & French (1988), Campbell & Shiller (1988), Stambaugh (1999), Campbell & Viceira (2005), textbooks) *vs* "*skeptics*" Ang & Bekaert (2007) and Boudoukh, Richardson & Whitelaw (2008):

  Stronger & less noisy signal *vs* cleansing away spurious persistence in residuals of predictive regressions due to overlapping data.

- Most literature centered on estimation & inference issues.

- We focus instead (as Kandel & Stambaugh (1996), Barberis (2000), Brennan & Xia (2010), Campbell & Viceira (2005)) on actual portfolio allocation: investor's problem is to select a *portfolio strategy*, not a *model*.

- Our purpose is to assess the economic significance of LHP from a **dynamic asset allocation perspective: how valuable is the information contained in LH returns for a rational (in particular LT, but not only) investor?**

- We differentiate between **prediction** horizon ***h*** and **investment** horizon ***T***.

- Most importantly, we use **truly LH (*t*+*h*) excess returns**, as opposed to **hypothetical** LH ones obtained by mere, mechanical forward recursion of Eq. for (t+1) excess returns, as always done in extant literature.
- We consider both stocks and bonds (plus a *stochastic* riskless asset)
- We measure the welfare (**certainty equivalent return rate**) associated with each optimal strategy across h and T
- [We assess the welfare loss associated with (2) sub-optimal strategies]
- [We construct rolling portfolios (to remedy single sample paths)]
- We construct **out-of-sample** portfolios (both optimal and myopic)

2. FRAMEWORK

➢The economy

Arbitrage-free, frictionless, incomplete market, continuous trading, **3 assets**:

$$dr_t = \theta_r \left[\overline{r} - r_t\right] dt + \sigma_r dZ_{r,t}. \tag{2}$$

$$\frac{dM_t}{M_t} - r_t dt = \mu_{M,t} dt + \sigma_M dZ_{M,t}, \tag{3}$$

$$\frac{dB_t}{B_t} - r_t dt = \mu_{B,t} dt + \sigma_B dZ_{B,t}, \tag{4}$$

$$\mu_{M,t} = \mu_{M,0} + \mu_{M,1} z_{1,t} + \mu_{M,2} z_{2,t}, \tag{5}$$

$$\mu_{B,t} = \mu_{B,0} + \mu_{B,1} z_{1,t} + \mu_{B,2} z_{2,t}. \tag{6}$$

The **2 predictors** [and r(t)] obey OU processes:

$$dz_{i,t} = \theta_i \left[\overline{z_i} - z_{i,t} \right] dt + \sigma_{z_i} dZ_{z_i,t}, \ \forall i = 1, 2, \tag{7}$$

Investor's program (standard CRRA):

$$\underset{\{\omega_t\}}{MaxE_t} \left[\frac{V_T^{1-\gamma}}{1-\gamma} \right]$$

$$s.t.$$

$$\frac{dV_t}{V_t} = r_t dt + \omega_{M,t} \left[\frac{dM_t}{M_t} - r_t dt \right] + \omega_{B,t} \left[\frac{dB_t}{B_t} - r_t dt \right], \tag{9}$$

The **simplest** model, to **focus** on issue of **LHP** (extensions are possible)

Explicit solution:

$$\begin{bmatrix} \omega_{M,t} \\ \omega_{B,t} \end{bmatrix} = \frac{1}{\gamma}\Sigma^{-1}\begin{bmatrix} \mu_{M,0} + \mu_{M,1} z_{1,t} + \mu_{M,2} z_{2,t} \\ \mu_{B,0} + \mu_{B,1} z_{1,t} + \mu_{B,2} z_{2,t} \end{bmatrix} \tag{10}$$
$$+ \frac{\left(A_1 (T-t) + A_{11}(T-t) r_t + A_5 (T-t) z_{1,t} + A_6 (T-t) z_{2,t}\right)}{\gamma}\Sigma^{-1}\Sigma_r$$
$$+ \frac{\left(A_2 (T-t) + A_{22}(T-t) z_{1,t} + A_4 (T-t) z_{2,t} + A_5 (T-t) r_t\right)}{\gamma}\Sigma^{-1}\Sigma_1$$
$$+ \frac{\left(A_3 (T-t) + A_{33}(T-t) z_{2,t} + A_4 (T-t) z_{1,t} + A_6 (T-t) r_t\right)}{\gamma}\Sigma^{-1}\Sigma_2.$$

- Mean-Variance term + 3 Intertemporal Hedging terms (Merton-Breeden)

- System of 10 Riccati ODEs solved numerically (as usual)

- **Need to estimate 27 parameters**

for a ***given*** investment horizon $\boldsymbol{T}$ and a ***given*** prediction horizon $\boldsymbol{h}$.

- **Then we can compute certainty equivalent (annualized) rates of return** ($\gamma = 2$) (more γs in Internet Appendix)

3. ESTIMATION

As data are discrete, use discretized version of continuous process for each *z*. Note that using 1 month as the discrete time step, as everyone does, is (theoretically) arbitrary and introduces an issue into the setting.

➢ **Procedure** for $h = 1(\Delta t)$:

- Integrate OU processes for z_i and r over $[t, t+\Delta t]$.
- Identify with discrete time regression parameters.

- Integrate processes for the stock and bond excess returns over $[t, t+\Delta t]$.
- Identify with discrete time regression parameters.

Procedure for h ≥ 2(Δt). One may consider 2 approaches (a third, later on).
Literature on asset allocation: infers LH processes from the SH process by simple forward recursion of the discretized regressions for the stock and bond excess returns: no information is gained by varying *h*. Consequently, one just has to estimate, *once and for all*, the coeffs for h=1(Δt) and then to generate mechanically (i.e. by known deterministic functions) the coefficients for *any h*, irrespective of what actual data say. For instance (stock market):

$$
\begin{aligned}
r_{M,t,t+h} - \int_t^{t+h} r_s ds = {} & \left[\frac{h}{\Delta t} . a_{M,\Delta t} + \sum_{i=1}^{2} a_{z_i,\Delta t} \beta_{M,\Delta t,i} \sum_{j=1}^{h/\Delta t-1} \left(\frac{h}{\Delta t} - j\right) A_{z_i,\Delta t}^{j-1} \right] \\
& + \left(\sum_{i=1}^{2} \beta_{M,\Delta t,i} \left[\sum_{j=0}^{h/\Delta t-1} A_{z_i,\Delta t}^{j} \right] \right) z_{i,t} \\
& + \left[\sum_{j=1}^{h/\Delta t} \varepsilon_{M,t+(j-1)\Delta t,t+j\Delta t} + \sum_{i=1}^{2} \beta_{M,\Delta t,i} \sum_{k=1}^{h/\Delta t-1} \upsilon_{z_i,t+(k-1)\Delta t,t+k\Delta t} \sum_{l=0}^{h/\Delta t-k-1} A_{z_i,\Delta t}^{l} \right],
\end{aligned}
$$

Literature on inference issues: uses truly observed LH returns (generally implies a SH return dynamics ≠ from explicit one); all relevant parameters, strategies and welfare, will genuinely depend on ***h***. We do that, but for asset allocation. For instance (stock market):

$$r_{M,t,t+h} - \int_t^{t+h} r_s ds = a_{M,h} + \beta_{M,h,1} z_{1,t} + \beta_{M,h,2} z_{2,t} + \upsilon_{M,t,t+h}$$

Note, now, the dependence of coefficients on ***h***.

Makes a crucial difference (see below Table 3).

➢Data and predictors

Long time span. Monthly data 1942:M1 to 2010:M12 (828 obs)

"Mkt" (CRSP files: index from NYSE, AMEX & NASDAQ stocks),

"Bond" (constant 20-y maturity),

"tb" (1-month T-Bill rate).

Predictors: dividend yield ***dy***, default spread (10-y Baa – Aaa) ***def***.

➢Predictive regressions for stock and bond markets

Overall: conventional finding.

As in Campbell, Lo & MacKinlay (1997), and Cochrane (2001), R^2s increase with ***h*** (mechanically, from 0.01 to 0.40 for stock market for ***h*** from 1 to 60 months).

We compute 2 *t*-stats: Newey-West (autocorrelation and heteroskedasticty) and (rather drastic) Hodrick (1992) (for Overlapping Data)

True also for the constant maturity bond.

But has that finding any economic meaning? In fact, NO.

PARAMETER ESTIMATION

- Parameters for predictor processes do depend strongly on ***h***

- Parameters for stock and bond excess returns:

Equity premium EP: Conditional volatility slightly increases (***h*** $\leq$ 1 year) then noticeably decreases, the latter result in line with raw data (unconditional volatility).

Bond premium BP: Conditional volatility is U-shaped, as is unconditional volatility in raw data.

⇔ **One cannot mechanically infer parameters from those of AR(h=1Δt).**

*** Estimated betas and volatilities of LH Excess Returns**

We present both results ("Realized" (us) vs "Hypothetical" (lit.)) in Table 3.

Table 3 – Panel A: Market excess returns *

		$\beta_{M,h,dy}$	$\beta_{M,h,def}$	**Cond. Std.**	**Unc. Std.**	**R^2**
Hypothetical	1m	**3,815**	**1,967**	**0,149**	**0,149**	**0,007**
	3m	3,770	1,912	0,147	0,149	0,021
	6m	3,703	1,834	0,144	0,147	0,040
	1y	3,574	1,690	0,140	0,146	0,077
	2y	3,333	1,445	0,133	0,143	0,138
	3y	3,114	1,248	0,127	0,141	0,185
	4y	2,914	1,087	0,123	0,139	0,220
	5y	**2,732**	**0,956**	**0,120**	**0,138**	**0,246**
Realized	1m	3,815	1,967	0,149	0,149	**0,007**
	3m	4,100	2,358	0,156	0,158	0,023
	6m	4,523	**3,577**	**0,158**	0,163	0,057
	1y	**4,604**	2,294	0,156	**0,165**	0,112
	2y	4,039	-1,65	0,141	0,155	0,173
	3y	3,655	**-2,32**	0,130	0,149	0,245
	4y	3,239	-1,95	0,120	0,143	0,297
	5y	**2,976**	**-1,40**	**0,115**	**0,141**	**0,338**

“Hypothetical”: smooth decrease in loadings and conditional std, even smoother decrease in unconditional std. Steady increase in R^2. (“Conventional academic wisdom”).

“Realized”: non monotonous variation in $\beta_{M,h,dy}$, in $\beta_{M,h,def}$ (which even changes sign!), and in both unconditional and conditional stds. Our R^2 increases more. Idem for bond returns.

Very strong indication of model misspecification.

4. PORTFOLIO ALLOCATIONS

Certainty equivalent rates of return (annualized) (Panel A Table 4)

2 clear patterns:

a) horizontally: “conventional professional wisdom”

b) vertically (more to our point): conventional academic wisdom, but is a little less obvious (since no absolute monotonicity)

** Table 4-Panel A: Optimal certainty equivalent (annualized)

	T=1m	3m	6m	1y	2y	5y	10y	30y
h=1m	**0.086**	**0.087**	**0.089**	**0.092**	**0.098**	**0.109**	**0.120**	**0.133**
3m	0.087	0.088	0.090	0.094	0.100	0.112	0.124	0.136
6m	0.088	0.090	0.093	0.098	0.106	0.122	0.135	0.146
1y	**0.083**	**0.085**	**0.089**	**0.095**	**0.105**	0.124	0.139	0.151
2y	0.083	0.087	0.092	0.102	0.117	0.144	0.164	0.182
3y	0.087	0.092	0.099	0.111	**0.132**	**0.168**	**0.193**	0.216
4y	0.091	**0.095**	**0.100**	**0.111**	0.129	0.164	0.192	**0.217**
5y	**0.091**	**0.093**	**0.097**	**0.104**	**0.117**	**0.146**	**0.172**	**0.197**

(average riskless rate is 4.1%).

Column-wise: Blue = Min, Red = Max.

◇ "Conventional professional wisdom": due to Sharpe ratio, CE increases as proportion of risky assets increases, when *T* increases. Conversely, as investor ages, *T* decreases, and this proportion declines and CE too.

◇ "Conventional academic wisdom": CE tends to increase with ***h***, for any given *T*. But humps do exist at some (*h, T*) couples: caution is required.

5. CORRECTING FOR OD

Valkanov (2003): correction for **persistence** in residuals of the predictive regressions of the stock and bond returns due to **return periods being larger than 1*Δt*** (for all ***h*** > 1 month) because of **overlapping data**.

Idea: estimates for slopes of predictive regressions for an asset excess return are consistent if we use "long period" values of the predictors. The latter are computed as the ***sum*** of the predictors' monthly values over the ***h***-long horizon.

$$r_{M,t+h} - \int_t^{t+h} r_s ds = a_{M,h} + \beta_{M,h,1} \sum_{i=0}^{h/\Delta t - 1} z_{1,t+i\Delta t} + \beta_{M,h,2} \sum_{i=0}^{h/\Delta t - 1} z_{2,t+i\Delta t} + \upsilon_{M,t+h}.$$

We redo the **whole exercise** with these new, in principle better, estimates.

*** Table 7 -A: Optimal certainty equivalent (annualized) - Valkanov

	T=1m	3m	6m	1y	2y	5y	10y	30y
h=1m	**0.086**	**0.087**	**0.089**	**0.092**	**0.098**	**0.109**	**0.120**	**0.133**
3m	0.097	0.099	0.101	0.104	0.109	0.119	0.126	0.136
6m	0.100	0.102	0.104	0.109	0.115	0.125	0.132	0.140
1y	**0.096**	**0.098**	**0.101**	**0.106**	**0.113**	**0.123**	**0.130**	**0.137**
2y	0.095	0.098	0.102	0.108	0.117	0.131	0.140	0.148
3y	0.100	0.103	0.107	0.114	0.125	0.142	0.153	0.163
4y	0.107	0.109	0.113	0.119	0.129	0.147	0.158	0.169
5y	**0.112**	**0.114**	**0.116**	**0.122**	**0.130**	**0.147**	**0.159**	**0.170**

Blue = Min, Red = Max (column-wise). Superiority of LHP even more convincing (Blue is always for *h* =1, Red for *h* = 60).

Conventional academic wisdom: No myth, but more complex than thought, as again monotonicity is not guaranteed (small slump for ***h*** = 1 year, across *T*).

Why? Investor benefits from information contained in the whole expected path followed by predictors, from t to $t+h$. One recovers the "strength" of the documented persistence of predictors.

→ **Predictability may be welfare improving** (although **R^2 is not** a relevant indicator, as portfolio allocation is dynamic).

→ What about the out-of-sample evidence?

6. OUT-OF-SAMPLE ANALYSIS

Almost a primer in AA: DeMiguel, Garlappi & Uppal (2009) is an exception, but they consider *static* MV portfolios only (so, essentially irrelevant here).

We estimate our parameters over the first 40 years (480 months) and construct OOS optimal portfolios for each (h,T) couple, with monthly (not "continuous", hence we are conservative) rebalancing as new predictor values are known but with the same estimated parameters for the whole investment period T. Full OOS sample thus spans 1982:M1 to 2010:M12 (336 months).

Beware: Non-annualized rates. Strategies are stopped when they deliver -100%. Also: inference problems OOS are ignored (since *relative* merit of LHP / SHP).

Table 9: Out-of-Sample Portfolios without Correction for OD

Panel B: Mean of the OOS return rates (optimal strategy)

	1m	**6m**	**1y**	**3y**	**5y**	**7y**	**10y**
1m	1.66%	1.53%	2.17%	6.94%	10.92%	6.69%	-35.92%
3m	0.60%	1.51%	2.50%	10.12%	24.96%	35.85%	-7.98%
6m	0.55%	1.41%	1.67%	8.04%	30.56%	44.25%	-7.14%
1y	0.54%	0.97%	0.46%	6.12%	25.55%	26.64%	-33.65%
2y	0.92%	3.83%	3.95%	21.01%	11.47%	-2.06%	-51.50%
3y	0.71%	1.93%	1.29%	22.49%	37.56%	35.00%	-51.28%
4y	0.46%	0.06%	-1.23%	11.42%	39.44%	26.44%	-28.03%
5y	0.11%	-1.12%	-2.76%	7.87%	20.73%	-1.59%	-66.33%

Poor results. LHP may look generally better than SHP, but with no pattern and often insignificant.

Panel C: *t*-stats for the mean difference in OOS rates

	1m	**6m**	**1y**	**3y**	**5y**	**7y**	**10y**
3m	-4.15	-0.03	0.46	3.14	8.95	15.98	11.57
6m	-3.51	-0.14	-0.42	0.55	5.52	9.19	7.34
1y	-2.27	-0.35	-0.79	-0.20	2.51	2.91	0.39
2y	-0.74	0.55	0.36	1.28	0.03	-0.47	-2.82
3y	-1.13	0.12	-0.22	1.46	0.63	0.69	-0.92
4y	-2.34	-0.84	-1.54	0.80	3.43	1.64	0.61
5y	-3.49	-1.80	-2.31	0.18	1.41	-0.81	-3.56

Mostly negative, except for LT investors and $h \leq 1$ year. In agreement with the skeptics.

However: Opposite conclusions with Valkanov's correction for OD

Table 10: Out-of-Sample Portfolios with Correction for OD (Full Sample)

Panel B: Mean of the OOS return rates (optimal strategy)

	1m	6m	1y	3y	5y	7y	10y
1m	1.66%	1.53%	2.17%	6.94%	10.92%	6.69%	-35.92%
3m	1.37%	4.07%	7.91%	28.46%	38.35%	34.11%	47.16%
6m	1.37%	4.60%	9.24%	36.72%	56.86%	68.11%	130.20%
1y	1.38%	4.75%	9.64%	36.66%	67.96%	112.67%	229.29%
2y	1.55%	5.71%	10.38%	27.42%	57.05%	78.22%	44.71%
3y	1.63%	6.37%	9.18%	16.18%	18.89%	21.52%	72.45%
4y	1.13%	4.14%	3.87%	2.99%	-8.66%	-9.23%	75.36%
5y	0.95%	3.55%	4.92%	-5.06%	-19.04%	-16.72%	53.25%

Much better results. In some cases, even outperforms the very-hard-to-beat (stock market) B&H strategy.

Average Turnovers of portfolios remain reasonable.

Still no monotonous relationship, however: no dream is afforded. Specification problem is pervasive.

Panel C: *t*-stats for the mean difference in OOS rates

	1m	**6m**	**1y**	**3y**	**5y**	**7y**	**10y**
3m	-0.77	3.39	4.88	6.73	5.91	6.80	17.70
6m	-0.59	3.05	4.49	7.09	7.53	10.06	15.50
1y	-0.53	2.55	3.85	6.36	8.15	10.86	15.91
2y	-0.16	1.98	2.78	3.94	5.65	5.82	7.93
3y	-0.04	1.52	1.70	1.33	0.70	1.07	2.62
4y	-0.52	0.75	0.38	-0.47	-1.43	-1.13	2.38
5y	-0.74	0.65	0.61	-1.64	-3.30	-1.80	1.77

LHP (up to 2 years, sometimes 4 years) better than SHP except for ST investors.

For a given *h*, *t*-stats increase monotonously with *T*

For a given *T*, inverted U-shaped pattern for *t*-stats

Financial crisis.

We redo for OOS sub-period ending in 2006:M12. We find:

Higher mean returns across the board, as expected. *t*-stats larger, with essentially same pattern as in full OOS period.

7. TAKE AWAY and ONGOING RESEARCH

- Unlike others (in the asset allocation literature), who infer ($t+h$) excess returns from “recursive” t+1 ones, we account directly for genuine ($t+h$) excess returns.
- There is both short run and long run (in-sample) predictability; but R^2s are not reliable indicators. In addition, they are essentially meaningless from the (economic) perspective of dynamic asset allocation
- Academic conventional wisdom seems, however, right in-sample: substantial gains from LH predictability at least for MLT investors.

- [Welfare loss fairly high for ignorant strategies, much less so for myopic ones.]
- [All the more so that *investment* and *prediction* horizons are large.]
- OOS analysis: without Valkanov correction, ARP is **illusory**. With it, LHP seems **valuable** for ***h*** between 6 months and 2-3 years, except for very short term (one-month) investors.
- [Taking the recent crisis into account or not does not change much the thrust of OOS results.]
- [Our results are robust to changes in predictors (EP ratio and stock market realized volatility) and in portfolios (Value and Growth).]

- We are currently trying to explore in more depth why our estimates obtained using LH linear projections are so often very different from those obtained from SH projections with the same data set.
- Models are intrinsically misspecified: there is very little hope to come up with a model that explains the data generating processes at short, medium and long horizons (especially using OD).
- We have, however, redone almost entirely the analysis with the GMM method, and computed the GMM estimates of our 27 parameters (for each ***h*** and ***T***). Overall, the previous results already seem vindicated in the sense that using the (arbitrarily selected) 1-month returns along with any longer term

returns generally beats (in CE terms) using 1-month returns only: LH returns seem to possess extra predictive power that should not be ignored.

- Another route to be explored is that economic agents are engaged in a learning process that induces both portfolio revisions and the reported differences between LH and SH estimates. Our preliminary results indicate that introducing learning does help a bit but is far from explaining the whole picture.