Fairness under risk and uncertainty

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Social choice under uncertainty

- Public policies involve dealing with risk/uncertainty: unemployment, health, environment.
- Key result in social choice theory: Harsanyi's (1955) theorem.
- ***Theorem:*** *(Expected utility + Pareto ex ante) imply that the social criterion must be a weighted sum of individuals' expected utilities.*

PROBLEMS WITH HARSANYI'S THEOREM

Harsanyi's result has serious drawbacks:

- Ex ante vs ex post equity: Diamond (1967); Broome (1991).
- Conflict between equity and Pareto in a multidimensional framework.
  Applied to risk: Gajdos and Tallon (2002); Fleurbaey and Maniquet (2011).
- Spurious unanimity and conflicting beliefs: Mongin (1995, 1998); Gilboa, Samet, Schmeidler (2004).
- Conflict between social rationality and Pareto for non-SEU models: Danan, Gajdos, Hill and Tallon (2014).

EX ANTE AND EX POST EQUITY

Consider the following lotteries (with $p(\omega_1) = p(\omega_2) = 1/2$):

	ω_1	ω_2
u_1	1	0
u_2	1	0

Lottery 1

	ω_1	ω_2
u_1	1	0
u_2	0	1

Lottery 2

	ω_1	ω_2
u_1	1	1
u_2	0	0

Lottery 3

Diamond's (1967) criticism: Lottery 2 is better than Lottery 3 because equal ex ante.
Broome's (1991) criticism: Lottery 1 is better than Lottery 2 because equal ex post.

Pareto vs. equity and social rationality

When individuals are SEU with same beliefs but different risk aversion, allocations of resources may seem unfair even though Pareto dominant:

	ω_1	ω_2
c_1	100	1000
c_2	100	1000

Lottery 1

	ω_1	ω_2
c_1	80	1100
c_2	110	800

Lottery 2

If $U_1 = \frac{1}{2}(c_{11})^{1/2} + \frac{1}{2}(c_{12})^{1/2}$ and $U_2 = -\frac{1}{2}(c_{21})^{-1/2} - \frac{1}{2}(c_{22})^{-1/2}$.

- When individuals have different beliefs and risk attitudes, it is impossible to consistently aggregate individual preferences (Hylland and Zeckhauser, 1979; Mongin, 1995).
- Several authors have argued that the Pareto principle is not appealing in this context (Gilboa, Samet, Schmeidler, 2003; Mongin, forth.).
- The main reason is what is called *spurious unanimity*: people may have convergent opinions based on conflicting (false) beliefs.

NON EXPECTED UTILITY AND SOCIAL RATIONALITY

- When individuals are not SEU, social assessment may violate weak rationality requirements even when individuals have exactly the same preferences.
- Consider the following example by Danan, Gajdos, Hill and Tallon (2014):
  2 individuals, 2 states, MEU preferences:

  $$U_i(f) = min_{p \in C}\, p u_i\big(f(\omega_1)\big) + (1-p) u_i\big(f(\omega_2)\big).$$

  If $u_1\big(f(\omega_1)\big) = u_2\big(f(\omega_2)\big) = 1$, $u_1\big(f(\omega_2)\big) = u_2\big(f(\omega_1)\big) = 0$, $u_1\big(g(\omega_1)\big) = u_2\big(g(\omega_2)\big) = u_1\big(g(\omega_2)\big) = u_2\big(g(\omega_1)\big) = 1/2$, and $u_0 = 1/2 u_1 + 1/2 u_2$,
  then $g \succ_1 f, g \succ_2 f$, while $u_0\big(g(\omega_1)\big) = u_0\big(g(\omega_2)\big) = u_0\big(f(\omega_1)\big) = u_0\big(f(\omega_2)\big)$.

Moral dilemmas

Main issues in the risk/uncertainty framework:

- Compatibility between equity and Pareto requirement.
- Individuals are not informed and may have different beliefs.
- Ex ante versus ex post evaluation.

Dilemma:

- One can endorse the Pareto principle, but to satisfy ex ante equity one has to abandon social rationality (expected utility; statewise dominance): Diamond (1967), Epstein and Segal (1992), Grant, Kajii, Polak, Safra (2010).
- One can seek to promote social rationality and ex post equity, but then one has to abandon the usual efficiency requirement: Fleurbaey (2010), Grant, Kajii, Polak, Safra (2012).

A variety of theories of justice under uncertainty

- There are experimental evidence that people hold a variety of views regarding justice in risky situations (Cappelen, Konow, Sorensen and Tungodden, 2013; Cettolin and Riedl, 2013):
  - Utilitarian/Harsanyian view.
  - Equalizing expected utility.
  - Ex ante equity: equalizing expected income.
  - Ex post equity: equalizing realized income...
- Views may depend on whether individuals took risk or were submitted to risk (Cappelen, Konow, Sorensen and Tungodden, 2013).
- In this talk, I will explore some solutions, in the context of a specific approach: fair social choice (Fleurbaey and Maniquet, 2011).

MEASURING WELFARE IN ECONOMIC ENVIRONMENTS

- Most of the literature following Harsanyi's theorem takes for granted that vNM utilities are the right metric for measuring individual wellbeing (Epstein and Segal, 1992; Ben Porath et al., 1997; Gajdos and Maurin, 2004...).
- The measurement of wellbeing is a longstanding issue in social choice and welfare theory.
- Fair social choice takes a specific approach in terms of measuring welfare: measures based on resources but using ordinal preferences.

Fair social choice

- Measurement of welfare based on resources: money-metric utility and equivalent income (Samuelson, 1974; King, 1983).

- Fair allocation theory and fair social choice often express fairness principles in terms of resources, e.g. using transfer principles (Pigou-Dalton).

- Fleurbaey and Maniquet (2011) have developed fair social choice theory. They only developed an ex ante approach in the case of risk.

FUTURE GENERATIONS AND VARIABLE POPULATION

Many issues involve future generations. This brings additional issues:

- The risk on the existence of future generations and discounting (Schelling, 1995; Stern, 2006; Nordhaus, 2007, 2008; Weitzman, 2007 Dasgupta, 2008).
- Preference diversity/ preference change: how can we compare individuals with different preferences?
- Preference uncertainty and the comparison of different populations: population ethics problem (Blackorby, Bossert and Donaldson, 2005; Asheim and Zuber, 2015).

FRAMEWORK

- Economy: n individuals denoted i; two goods, market good $c \in \mathbb{R}_+$ and non-market good $q \in [0,1]$.
  A bundle for individual i is $x_i = (c_i, q_i)$.
  An allocation: $x = (x_i)_{i \in N}$. $X = (\mathbb{R}_+ \times [0,1])^n$ the set of allocations.
- Uncertainty: Savage framework.
  - $\omega \in \Omega$: state of the world.
  - $f : \Omega \to X$, a simple act. $\mathcal{F}$ the set of simple acts.
    $f = (f_1, \cdots, f_n)$.
  - $x \in \mathcal{F}$, a constant act.

Individual Preferences

Each individual i has preferences satisfying the following conditions:

- Preferences are represented by a complete preorder R_i over $(\mathbb{R}_+ \times [0,1])^{\Omega}$.
- There exists a continuous, increasing and quasi-concave function $u_i : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ such that:

$$f_i \, R_i \, g_i \iff \mathbb{E}\big(u_i(f_i)\big) \geq \mathbb{E}\big(u_i(g_i)\big).$$

- For some $\bar{q}$, and for any $x_i = (c_i, q_i)$ there exists $z \in \mathbb{R}_+$ such that $u_i(z, \bar{q}) = u_i(c_i, q_i)$.

Equivalent market good consumption

Definition

For any $x_i \in \mathbb{R}_+ \times [0,1]$, and given the preferences R_i of individual i, the *equivalent market good consumption* of x_i, denoted $e_i(x_i)$, is the positive scalar such that

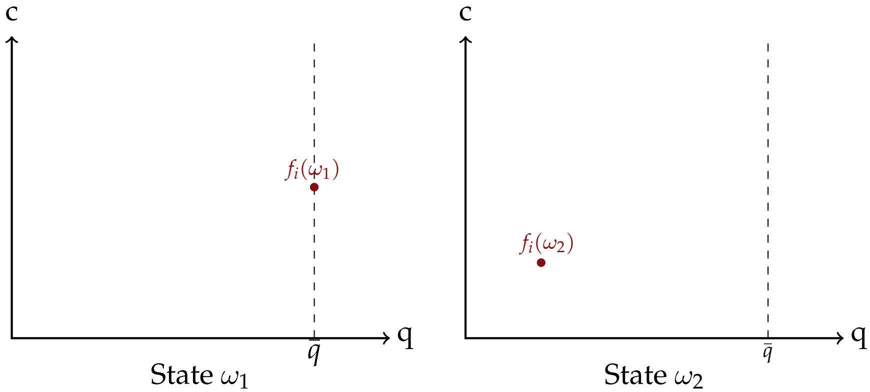
$$(e_i(x_i), \bar{q}) \; I_i \; x_i.$$

Definition

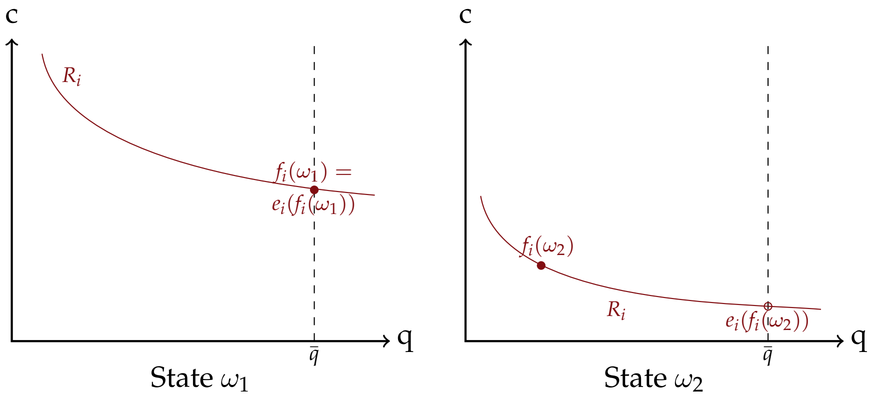
For any $f_i \in \left(\mathbb{R}^2_+\right)^\Omega$, and given the preferences R_i of individual i, the *certainty equivalent market good consumption* of f_i, denoted $ce_i(f_i)$ is the positive scalar such that

$$(ce_i(f_i), \bar{q}) \; I_i \; f_i.$$

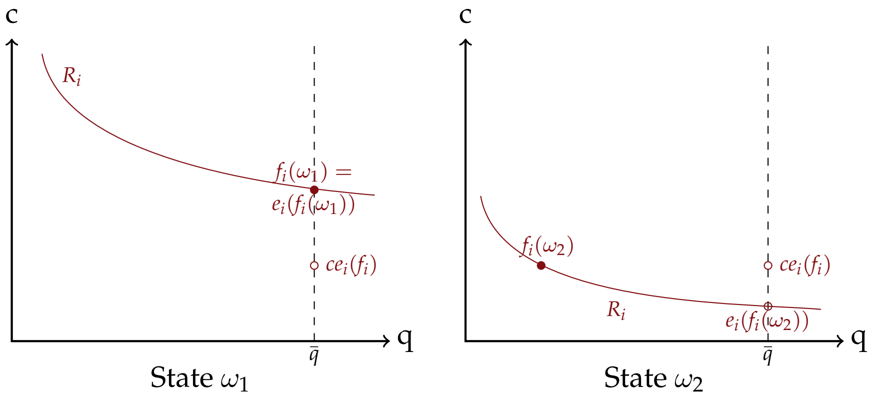
GRAPHICAL ILLUSTRATION



Graphical Illustration



Graphical Illustration



Certainty equivalent consumption as a welfare measure

- The equivalent market good consumption level $e_i(x_i) \in \mathbb{R}_+$ is a proper welfare measure, which is ordinally equivalent to any individual utility function.
- There exists a continuous and increasing function v_i such that $u_i(x_i) = v_i\big(e_i(x_i)\big)$
- Also:

$$ce_i(f_i) = v_i^{-1}\Big(\mathbb{E}\big(u_i(f_i)\big)\Big),$$

is a measure of ex ante welfare.

Social orderings

- R over acts (uncertain allocations).
- R^0 over allocations.
  - R^0 is based on an ordering R^* on $\mathbb{R}_+$, so that $\forall x, y \in X$,

$$xR^0y \Leftrightarrow (e_i(x_i))_{i \in N} \, R^* \, (e_i(y_i))_{i \in N} \, .$$

  - R^* is supposed monotonic, quasi-concave (equity) and continuous (for the presentation).

Equally-Distributed Equivalent (EDE)

- X^e: egalitarian allocations, such that $e_i(x_i) = e_j(x_j)$.
  $\mathcal{F}^e$: egalitarian acts.

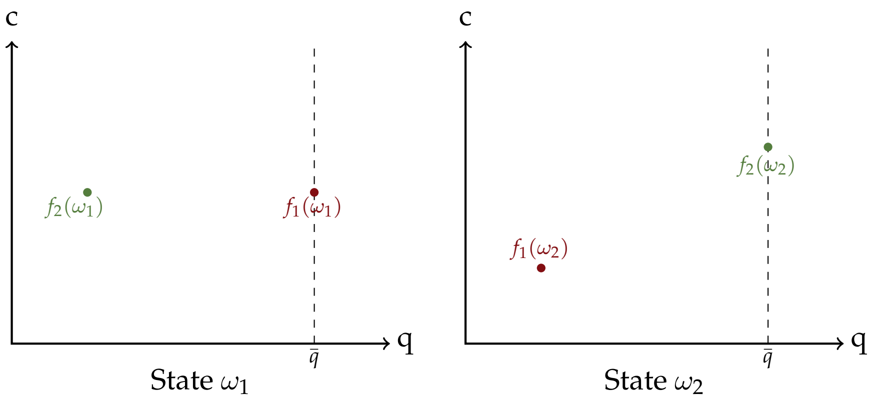
- ***Definition:*** For any $x \in X$, $x^e \in X^e$ is an EDE allocation if,

$$xI^0x^e.$$

  An EDE act $f^e \in \mathcal{F}^e$ is an act yielding an EDE allocation in each state of the world.

- The concept of an EDE can be extended when R^0 is not continuous: Equally-Distributed Quasi-Equivalent (EDQE).

Graphical Illustration



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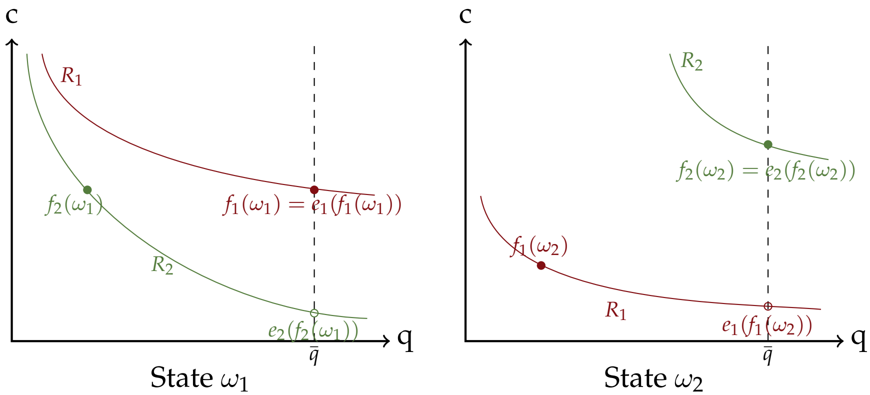

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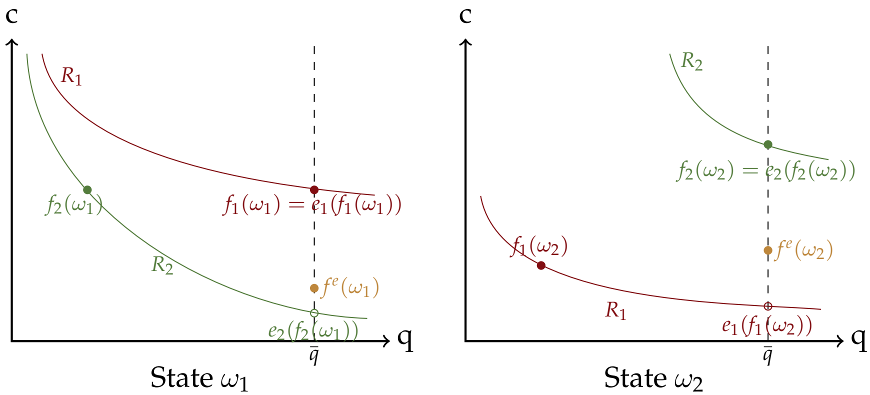

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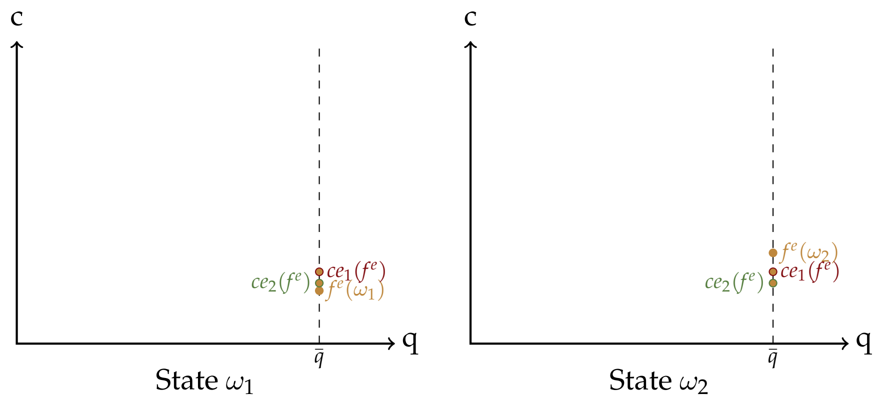


Ex ante versus ex post approaches

- Even in simple case with one consumption good, two equally probable states of the world and two individuals with different risk preferences, there exist a direct conflict between the Pareto principle and Statewise Dominance for fair social criteria.
- If the two individuals have different preferences there exist levels of consumption c, $\bar{c}$ and $\underline{c}$ such that individual 1 prefers the sure level c to a prospect of having each of $\bar{c}$ and $\underline{c}$ with probability 1/2, while individual 2 prefers the uncertain prospect to the sure consumption.
- Dilemma when the society may face the following prospects

$$\begin{pmatrix} c & c \\ \bar{c} & \underline{c} \end{pmatrix} \text{ and } \begin{pmatrix} \bar{c} & \underline{c} \\ c & c \end{pmatrix}.$$

THE EX ANTE APPROACH

The ex ante approach will abandon Statewise Dominance (where social preference R_0 are defined by a fair criterion) to satisfy the Pareto principle:

Axiom: Pareto

For $f, g \in \mathcal{F}$, if $f_i R_i g_i$ (resp. $f_i P_i g_i$) for all $i \in N$, then fRg (resp. fPg).

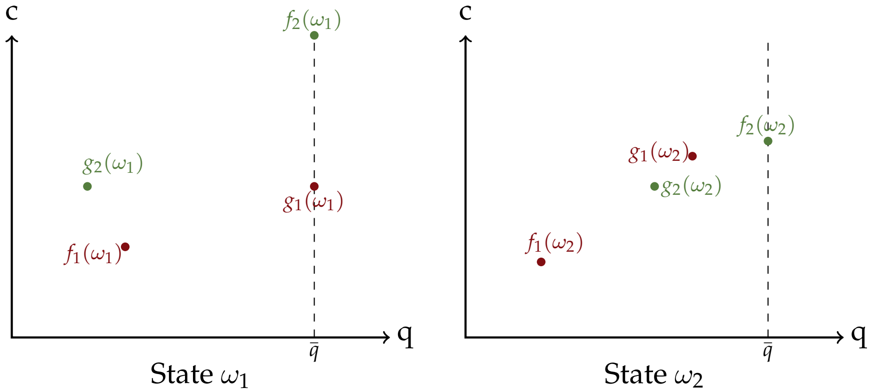
We have the following result:

Proposition

If R^0 is continuous and R satisfies Pareto, then fRg if and only if

$$ce(f) \; R^0 \; ce(g).$$

Proof


c
$f_2(\omega_1)$
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State ω_1
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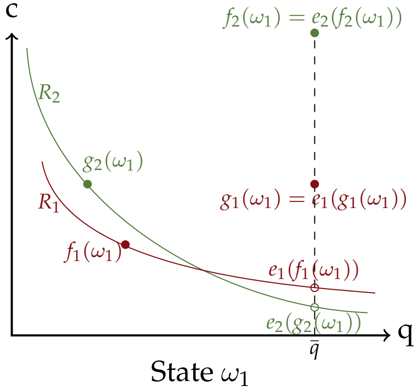
State ω_2

Proof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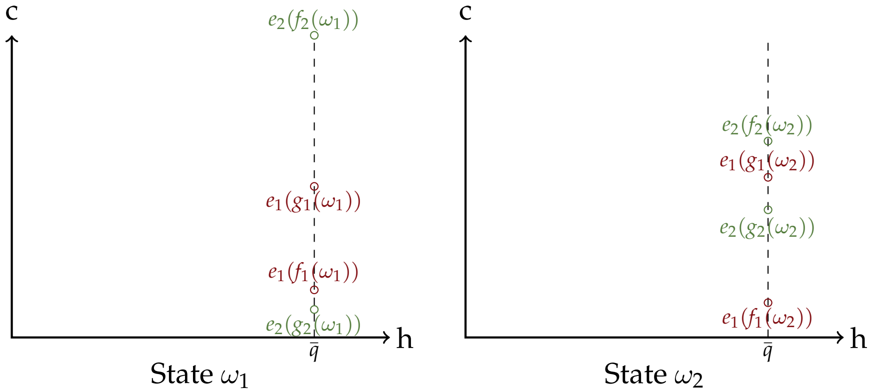
Proof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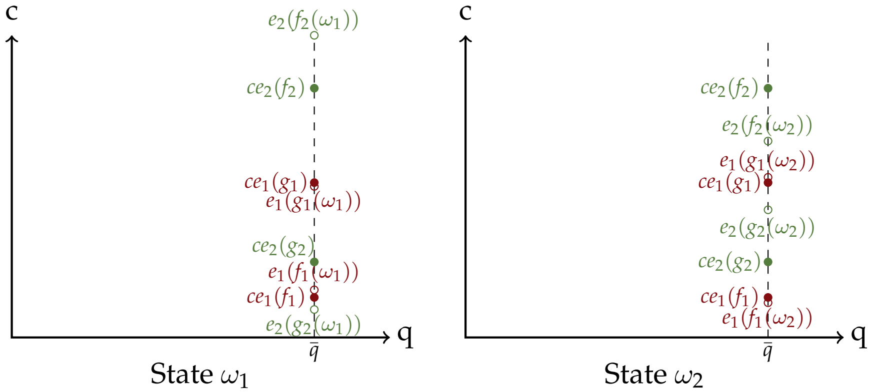
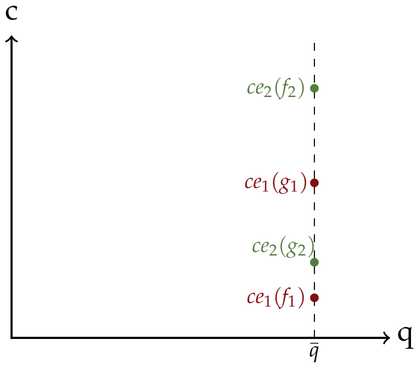
State ω_2

Proof



$$f \; R \; g \iff ce(f) \; R \; ce(g) \qquad (Pareto)$$

Proof



$$
\begin{aligned}
f\,R\,g &\iff ce(f)\;R\;ce(g) && (Pareto)\\
&\iff ce(f)\;R^0\;ce(g)
\end{aligned}
$$

Ex ante criteria: comments

- Implicit assumption: R^0 is used to rank sure allocations. The result is a direct application of the Pareto principle.
- It may be possible to add constraints on what R^0 should be:
  - For instance, if we want the social criterion to be an expected utility, we will obtain a linear aggregation of some VNM utility functions (but these are generally not the welfare metrics of the fair social choice approach).
  - Fleurbaey and Maniquet (2011) added an independence condition to obtain that R^0 is a maxmin.

Ex post Axioms

For the ex post approach, consider the following axioms:

Axiom: Dominance

For $f, g \in \mathcal{F}$, if for all $\omega \in \Omega, f(\omega) R^0 g(\omega)$ (resp. $f(\omega) P^0 g(\omega)$) then fRg (resp. fPg).

Axiom: Pareto for Equal or No Risk

For $f, g \in (\mathcal{F}^e \cup X)$, if $f_i R_i g_i$ (resp. $f_i P_i g_i$) for all $i \in N$, then fRg (resp. fPg).

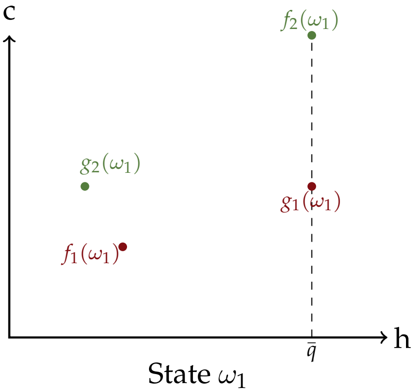
Proposition

Proposition

If R^0 is continuous and R satisfies Pareto for Equal or No Risk and Dominance, then fRg if and only if

$$ce\left(f^e\right) R^0 \, ce\left(g^e\right).$$

Proof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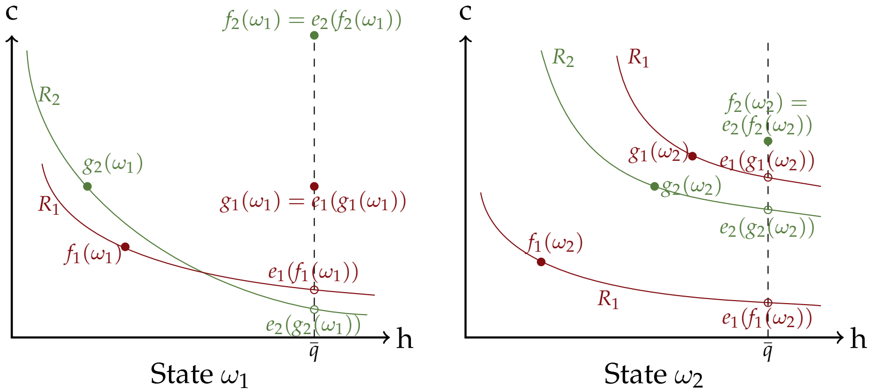
Proof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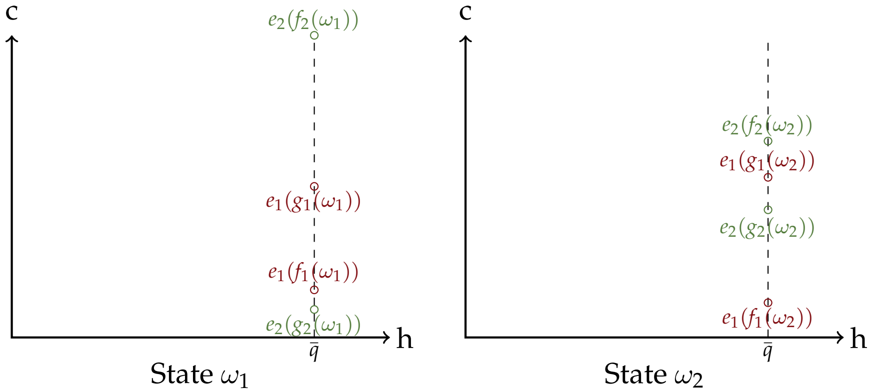
State ω_2

Proof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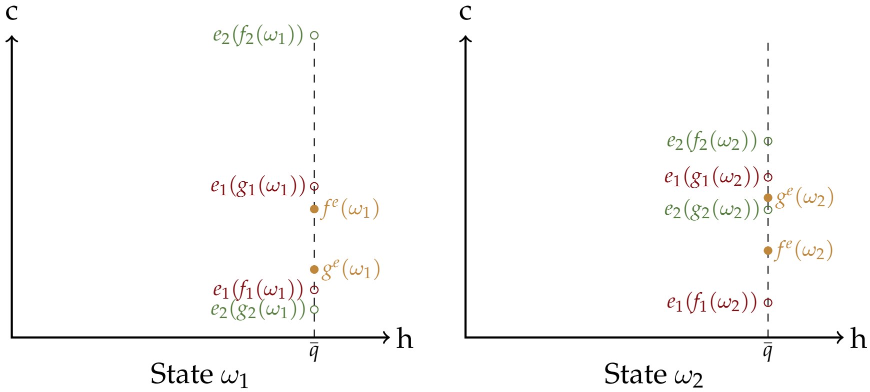
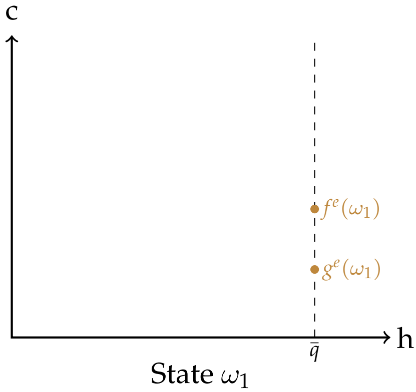
State ω_2

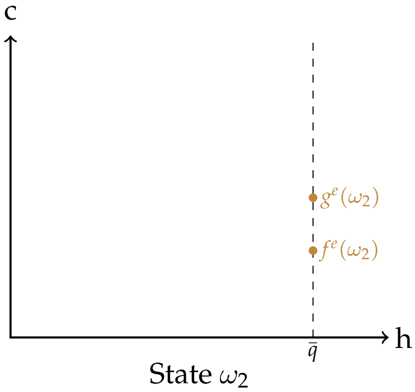
PROOF



$$f \; R \; g \iff f^e \; R \; g^e \qquad \textit{(Dominance)}$$

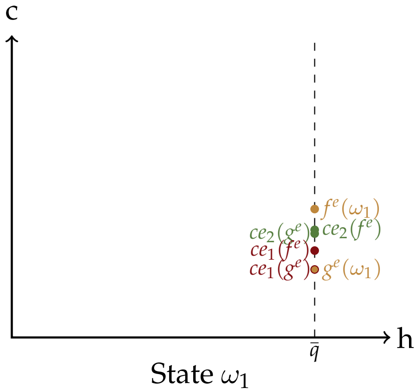
Proof





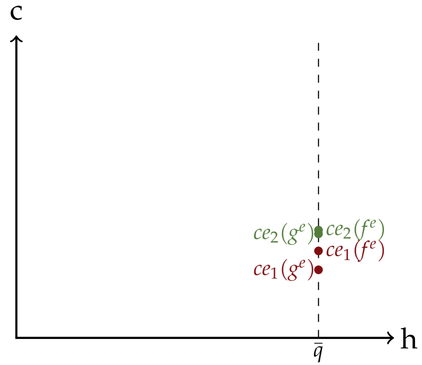
$$f \; R \; g \quad \Longleftrightarrow \quad f^e \; R \; g^e \qquad \textit{(Dominance)}$$

Proof



$$
\begin{aligned}
f\,R\,g &\iff f^e\,R\,g^e && \textit{(Dominance)}\\
&\iff ce(f^e)\,R\,ce(g^e) && \textit{(PENR)}
\end{aligned}
$$

Proof



$$
\begin{aligned}
f\ R\ g &\iff f^e\ R\ g^e && (Dominance)\\
&\iff ce(f^e)\ R\ ce(g^e) && (PENR)\\
&\iff ce(f^e)\ R^0\ ce(g^e) && (Dominance)
\end{aligned}
$$

EX POST CRITERIA: COMMENTS

- Ex post criteria are derived from natural properties:
  - Dominance is an appealing axiom of social rationality (cf. Monotonicity in the MBA model by Cerreia-Vioglio et al. 2011).
  - Fleurbaey (2010) proposed Pareto for Equal Risk. Justification: situations where individuals have same information as the society (equal risk).
    We strengthen Pareto for Equal Risk to add situations without risk (Pareto ex post).

- Ex post criteria have a three-steps structure:
  1. compute EDE in each state of the world;
  2. compute certainty-equivalent of the EDEs for all individuals;
  3. assess the distribution of these certainty-equivalent allocations using the social ordering for *ex post* allocations.

Fair criteria are not expected utilities

The two Propositions characterize criteria that generically are not expected utilities, even when individuals are expected utility maximizers.

Recall that there exists u_i such that for all $(c_i, h_i) \in \mathbb{R}_+ \times [0,1]$, $u_i(c_i, h_i) = v_i\Big(e_i\big(c_i, h_i\big)\Big)$, and

$$f_i R_i g_i \iff \int_\Omega u_i\big(f_i(\omega)\big)dp(\omega) \geq \int_\Omega u_i\big(g_i(\omega)\big)dp(\omega).$$

The concavity of v_i expresses risk aversion with respect to equivalent consumption.

An example

Assume that

$$
\begin{aligned}
xR^0y &\Leftrightarrow \left(e_i(x_i)\right)_{i\in N} R^* \left(e_i(y_i)\right)_{i\in N} \\
&\Leftrightarrow \sum_{i\in N} \psi(e_i(x_i)) \geq \sum_{i\in N} \psi(ce_i(y_i)).
\end{aligned}
$$

And denote $c^e(x)$ the consumption level

$$
c^e(x) = \psi^{-1}\left(\frac{1}{N}\sum_{i\in N} \psi(e_i(x_i))\right),
$$

i.e. $x^e = (c^e(x), 1)$.

An example

Then using the Propositions:

- In the ex ante approach:

$$fRg \iff \sum_{i\in N} \psi \circ v_i^{-1}\left(\int_\Omega v_i\Big(e_i\big(f_i(\omega)\big)\Big)dp(\omega)\right) \geq \sum_{i\in N} \psi \circ v_i^{-1}\left(\int_\Omega v_i\Big(e_i\big(g_i(\omega)\big)\Big)dp(\omega)\right)$$

- In the ex post approach:

$$fRg \iff \sum_{i\in N} \psi \circ v_i^{-1}\left(\int_\Omega v_i\Big(c^e\big(f(\omega)\big)\Big)dp(\omega)\right) \geq \sum_{i\in N} \psi \circ v_i^{-1}\left(\int_\Omega v_i\Big(c^e\big(g(\omega)\big)\Big)dp(\omega)\right)$$

AN EXAMPLE

The non separability issue can be resolved in the case where ψ is extremely concave (min), and $v_i(c) = c^{1-\gamma_i}/(1-\gamma_i)$.
Then, in the ex post approach (ex ante is similar):

$$
\begin{aligned}
fRg \quad &\Longleftrightarrow \quad \min_{i\in N}\left(\int_\Omega \left(\min_{j\in N} e_j(f_j(\omega))\right)^{1-\gamma_i} dp(\omega)\right)^{\frac{1}{1-\gamma_i}} \geq \\
&\qquad \min_{i\in N}\left(\int_\Omega \left(\min_{j\in N} e_j(g_j(\omega))\right)^{1-\gamma_i} dp(\omega)\right)^{\frac{1}{1-\gamma_i}} \\
&\Longleftrightarrow \quad \frac{1}{1-\gamma_{\max}} \int_\Omega \left(\min_{j\in N} e_j(f_j(\omega))\right)^{1-\gamma_{\max}} dp(\omega) \geq \\
&\qquad \frac{1}{1-\gamma_{\max}} \int_\Omega \left(\min_{j\in N} e_j(g_j(\omega))\right)^{1-\gamma_{\max}} dp(\omega).
\end{aligned}
$$

where $\gamma_{\max} = \max_{i\in N} \gamma_i$.

PENR, Fairness and Expected Utility

Domain Restriction: Existence of a more risk averse agent

There exists an individual such that, for any egalitarian act, her certainty-equivalent is lower than (or equal to) the one of any other individual.

Axiom: Social Expected Utility

For $f, g \in \mathcal{F}$, there exists $u : X \to \mathbb{R}$ such that

$$f \succeq g \iff \int_\Omega u(f(\omega))dp(\omega) \geq \int_\Omega u(g(\omega))dp(\omega).$$

Axiom: Independence of risk attitudes

The ex post social ordering R_0 is independent of the v_i functions (it only depends on ordinal information about individual preferences over sure allocations).

PENR, Fairness and Expected Utility

Proposition

On the domain of preferences satisfying Existence of a more risk averse agent, R satisfies Pareto for Equal and No Risk, Social Expected Utility and R^0 satisfies Independence of risk attitudes if and only R^0 is maxmin. Thus fRg if and only

$$E(u_k \circ f_i^e) \geq E(u_k \circ g_i^e),$$

where k is the most risk-averse agent

Non expected utility and beliefs

- Results of our main Propositions can obviously be extended to non expected utility models: we only need the existence of certainty equivalents.
- Problem: individuals may have different beliefs and spurious unanimities may arise.
- There have been several papers about how beliefs should be aggregated (Gilboa, Samet Schmeidler, 2004; Qu, 2014; Danan, Gajdos, Hill, Tallon, 2014).

Maximin expected utility

- Assume that all individuals are maxmin utility maximizers: for all i there exists a set C_i of probabilities over Ω such that:

$$f_i \succeq g_i \iff \min_{p \in C_i} \int_\Omega u\big(f_i(\omega)\big)dp(\omega) \geq \min_{p \in C_i} \int_\Omega u\big(g_i(\omega)\big)dp(\omega).$$

- Note that we assume that all individuals have the same function u.
- In general, it is not clear that the criterion defined in the Propositions should be a maxmin expected utility.
- Social preferences will define how beliefs should be aggregated.

THE MIN MAXIMIN EXPECTED UTILITY

- If R^0 is a maxmin, the obtain the following criterion:

$$f \succeq g \iff \min_{i \in \mathbb{N}} \left(\min_{p \in C_i} \int_\Omega u\left(\min_{j \in \mathbb{N}} ce_j(f_j(\omega)) \right) dp(\omega) \right) \geq \min_{i \in \mathbb{N}} \left(\min_{p \in C_i} \int_\Omega u\left(\min_{j \in \mathbb{N}} ce_j(g_j(\omega)) \right) dp(\omega) \right)$$

- This is equivalent to

$$f \succeq g \iff \min_{p \in C} \int_\Omega u(f(\omega)) dp(\omega) \geq \min_{p \in C} \int_\Omega u(g_i(\omega)) dp(\omega),$$

where $C = \cup_{i \in \mathbb{N}} C_i$.

- The social ordering exhibits more aversion to uncertainty than individual preferences.

FRAMEWORK

- Two goods: market good $c \in \mathbb{R}_+$ and non-market good $q \in [0,1]$.
  A bundle for individual i is $x_i = (c_i, q_i)$.

- Set of *potential* individuals is $\mathbb{N}$. In any particular alternative, a subset $\mathcal{N}$ of individuals exist. An allocation is $x \in \mathcal{X} = \bigcup_{\mathcal{N}} (\mathbb{R}_+ \times [0,1])^{\mathcal{N}}$.

- Uncertainty: Savage framework.
  - $\omega \in \Omega$: state of the world. $\mathcal{A}$ a sigma-algebra on Ω. $A \in \mathcal{A}$ an event.
  - $f : \Omega \to \mathcal{X}$, a prospect (simple act). $\mathcal{F}$ the set of prospects.
  - $x \in \mathcal{X}$, a sure prospect.

- For any $x \in \mathcal{X}$, $\mathcal{N}(x)$ the set of individuals alive, and $n(x) = |\mathcal{N}(x)|$. Similarly, for any $f \in \mathcal{F}$, for any $\omega \in \Omega$, we can define $\mathcal{N}(f(\omega))$ and $n(f(\omega))$.

Individual Preferences

Each individual i has preferences satisfying the following conditions:

- Preferences are represented by a complete preorder $\succeq_i$ over $\bigcup_{A \in \mathcal{A}} \left(\mathbb{R}_+^2\right)^A$.
- For any $f \in \mathcal{F}$, let $A_i(f) = \{\omega | i \in \mathcal{N}(f(\omega))\}$, and f_i a mapping from A_i to $\mathbb{R}_+ \times [0,1]$ assigning to i her allocation in $\omega \in A_i$ according to act f. Also denote $p_i(f) = p(A_i(f))$.
- There exists a continuous, increasing and quasi-concave function $u_i : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ such that:

$$f_i \succeq_i g_i \iff \mathbb{E}_A\left(u_i(f_i)\right) \geq \mathbb{E}_A\left(u_i(g_i)\right).$$

- For some $\bar{q}$, and for any $x_i = (c_i, q_i)$ there exists $z \in \mathbb{R}_+$ such that $u_i(z, \bar{q}) = u_i(c_i, q_i)$.

Ex ante approach: Axioms

1. Pareto
2. Anonymous Pigou-Dalton
3. Existence independence
4. Restricted expected utility
5. Replacement

Ex ante approach: Result

Proposition

If the social ordering satisfies Pareto, Anonymous Pigou-Dalton, Existence independence, Restricted expected utility and Replacement, then there exists $\alpha \in \mathbb{R}$ and an increasing continuous and concave function $\phi : \mathbb{R}_+ \to \mathbb{R}$ such that for all $f, g \in \mathcal{F}$:

$$f \, R \, g \Longleftrightarrow \sum_{i \in \mathbb{N}} p_i(f) \left[\phi(ce_i(f_i)) - \alpha \right] \geq \sum_{i \in \mathbb{N}} p_i(g) \left[\phi(ce_i(g_i)) - \alpha \right].$$

Ex post approach: Axioms

1. Pareto for no risk
2. Pareto for equal risk
3. Anonymous Pigou-Dalton
4. Separability for sure prospects
5. Social Expected Utility Hypothesis

Ex post approach: Result

Proposition

If the social ordering satisfies Pareto for no risk, Pareto for equal risk, Anonymous Pigou-Dalton, Independence for sure prospects and Expected utility and Replacement, then there exists an increasing continuous and concave function $\phi : \mathbb{R}_+ \to \mathbb{R}$ and for each $\mathcal{N}$ a vector of scalars $(\alpha_i^{\mathcal{N}})_{i \in \mathcal{N}}$ such that for all $f, g \in \mathcal{F}$:

$$f \, R \, g \iff \mathbb{E}\left(\sum_{i \in \mathcal{N}(f)} \alpha_i^{\mathcal{N}(f)} v_i \circ \phi^{-1}\left(\frac{1}{n(f)} \sum_{i \in \mathcal{N}(f)} \phi(e_i(f_i))\right)\right)$$

$$\geq \mathbb{E}\left(\sum_{i \in \mathcal{N}(g)} \alpha_i^{\mathcal{N}(g)} v_i \circ \phi^{-1}\left(\frac{1}{n(g)} \sum_{i \in \mathcal{N}(g)} \phi(e_i(g_i))\right)\right).$$

Conclusion

- We have developed fairness methods for analyzing economic policy in the case of uncertainty, including uncertainty about future preferences:
  - Ex ante approaches take concave transformations of certainty equivalent consumption (weighted by people probability of existence.)
  - Ex post approaches are expected values of equally distributed equivalent consumptions.
- The methods provide an explicit methodology for measuring and comparing welfare. They take different paths regarding when individual welfare should be aggregated.
- The social criteria are not necessarily expected utilities (and it is not always straightforward to link them to existing decision models). But there are interesting special cases (in particular maxmin cases).

Further questions / future directions

- How should we aggregate beliefs? Not clear, because our framework is different from most recent works on the topic, which use the Anscombe-Aumann framework and therefore linear aggregation under certainty (Cres, Gilboa and Vieille, 2011; Qu, 2014, Danan, Gajdos, Hill and Tallon, 2014).
- How should we normalize the risk functions v_i in the ex post approach? How should we choose the associated weights?
- Population ethics: our criteria do not avoid all issues (Repugnant Conclusion, Reverse Repugnant Conclusion, Sadistic Conclusion...).

Pareto

▸

Axiom: Pareto

For all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $\mathbf{x}_i \succeq_i \mathbf{y}_i$ for all $i \in \mathcal{N}$, then $\mathbf{x} \succeq \mathbf{y}$. If furthermore there exists $j \in \mathcal{N}$ such that $\mathbf{x}_j \succ_j \mathbf{y}_j$, then $\mathbf{x} \succ \mathbf{y}$.

Anonymous Pigou-Dalton

Axiom: Anonymous Pigou-Dalton

For all $x, y \in \bar{\mathcal{X}}$ such that $n(x) = n(y)$, if there exists a bijection $\pi : \mathcal{N}(x) \to \mathcal{N}(y)$, $i, j \in \mathcal{N}(x)$ and $\varepsilon > 0$ such that

1. $y^1_{\pi(i)} + \varepsilon = x^1_i \leq x^1_j = v^1_{\pi(j)} - \varepsilon$;
2. $x_k = y_{\pi(k)}$ for all $k \in (\mathcal{N}(x) \setminus \{i, j\})$,

then $x \succ y$.

Existence Independence

Axiom: Existence Independence

For all $\mathcal{M} \in \mathfrak{N}$, for all $\mathbf{x}, \tilde{\mathbf{x}}, \mathbf{y}, \tilde{\mathbf{y}} \in \mathcal{X}$, if
$\mathcal{S}_i(\mathbf{x}) = \mathcal{S}_i(\mathbf{y})$ and $\mathcal{S}_i(\tilde{\mathbf{x}}) = \mathcal{S}_i(\tilde{\mathbf{y}}) = \emptyset$ for all $i \in \mathcal{M}$,
$\mathcal{S}_i(\mathbf{x}) = \mathcal{S}_i(\tilde{\mathbf{x}})$ and $\mathcal{S}_i(\mathbf{y}) = \mathcal{S}_i(\tilde{\mathbf{y}})$ for all $i \in \mathbb{N} \setminus \mathcal{M}$,
$\mathbf{x}_i(s) = \mathbf{y}_i(s)$ for all $i \in \mathcal{M}$ and $s \in \mathcal{S}_i(\mathbf{x})$,
$\mathbf{x}_j(s) = \tilde{\mathbf{x}}_j(s)$ for all $j \in \mathbb{N} \setminus \mathcal{M}$ and $s \in \mathcal{S}_j(\mathbf{x})$,
$\mathbf{y}_j(s) = \tilde{\mathbf{y}}_j(s)$ for all $j \in \mathbb{N} \setminus \mathcal{M}$ and $s \in \mathcal{S}_j(\mathbf{x})$,
then $x \succeq y \iff \tilde{x} \succeq \tilde{y}$.

RESTRICTED EXPECTED UTILITY

Axiom: Restricted Expected Utility

There exists a continuous function $\tilde{U} : \mathcal{X} \to \mathbb{R}$ such that, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if there exists two partitions of $\mathcal{S}$ $(A_1, \cdots, A_m)$ and $(B_1, \cdots, B_n)$ such that $\mathbf{x}(s) = \mathbf{x}(s')$ for all $s, s' \in A_l$ $(l = 1, \cdots, m)$, $\mathbf{y}(s) = \mathbf{y}(s')$ for all $s, s' \in B_k$ $(k = 1, \cdots, n)$, $\mathcal{N}(\mathbf{x}(s)) \cap \mathcal{N}(\mathbf{x}(s')) = \emptyset$ for all $s \in A_l$ and $s' \in A_k$, $l \neq k$, and $\mathcal{N}(\mathbf{y}(s)) \cap \mathcal{N}(\mathbf{y}(s')) = \emptyset$ for all $s \in B_l$ and $s' \in B_k$, $l \neq k$:

$$\mathbf{x}R\mathbf{y} \iff \mathbb{E}\big(\tilde{U}(\mathbf{x})\big) \geq \mathbb{E}\big(\tilde{U}(\mathbf{y})\big)$$

REPLACEMENT

▸

Replacement

For all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if there exist $i, j, h \in \mathbb{N}$ such that:

- for all $r \in \mathbb{N} \setminus \{i, j, h\}$, $\mathbf{x}_r(s) = \mathbf{y}_r(s)$ for all $s \in A_r$;
- $\mathcal{S}_h(\mathbf{x}) = \emptyset$ and $\mathcal{S}_i(\mathbf{y}) = \mathcal{S}_j(\mathbf{y}) = \emptyset$;
- there exists $z \in \mathbb{R}_+$ such that $\mathbf{x}_i(s) = (z, \bar{x}^2)$ for all $s \in \mathcal{S}_i(\mathbf{x})$, $\mathbf{x}_j(s) = (z, \bar{x}^2)$ for all $s \in \mathcal{S}_j(\mathbf{x})$, and $\mathbf{x}_h(s) = (z, \bar{x}^2)$ for all $s \in \mathcal{S}_h(\mathbf{y})$;
- $p_h(\mathbf{y}) = p_i(\mathbf{x}) + p_j(\mathbf{x})$;

then $\mathbf{x} I \mathbf{y}$.

Pareto for no risk

-

Axiom: Pareto for no risk

For all $x, y \in \mathcal{X}$ if $\mathcal{N}(x) = \mathcal{N}(y) = \mathcal{N}$ and $x_i \succeq_i y_i$ for all $i \in \mathcal{N}$ then $x \succeq y$. If furthermore $x_j \succ_j y_j$ for some $j \in \mathcal{N}$ then $x \succ y$.

PARETO FOR EQUAL RISK

▸

Axiom: Pareto for equal risk

For all $\mathbf{x}, \mathbf{y} \in \bar{\mathbf{X}}$, if there exists $A \subset \mathcal{S}$ and $\mathcal{N} \in \mathfrak{N}$ such that $N(\mathbf{x}(s)) = N(\mathbf{y}(s)) = \mathcal{N}$ for all $s \in A$, $N(\mathbf{x}(s)) = N(\mathbf{y}(s)) \neq \mathcal{N}$ for all $s \in \mathcal{S} \setminus A$, and:

- for all $i, j \in \mathcal{N}$, $\mathbf{x}_i(s) = \mathbf{x}_j(s)$ and $\mathbf{y}_i(s) = \mathbf{y}_j(s)$ for all $s \in A$;
- $\mathbf{x}(s) = \mathbf{y}(s)$ for all $s \in \mathcal{S} \setminus A$;

and if for all $i \in \mathcal{N}$, $\mathbb{E}_A\big(u_i(\mathbf{x}_i)\big) \geq \mathbb{E}_A\big(u_i(\mathbf{y}_i)\big)$ then $\mathbf{x} \succeq \mathbf{y}$. If furthermore $\mathbb{E}_A\big(u_j(\mathbf{x}_j)\big) > \mathbb{E}_A\big(u_j(\mathbf{y}_j)\big)$ for some $j \in \mathcal{N}$, then $\mathbf{x} \succ \mathbf{y}$.

SEPARABILITY FOR SURE PROSPECTS

▸

Axiom: Separability for sure prospects

For all $\mathcal{M}, \mathcal{N} \in \mathfrak{N}$ such that $\mathcal{M} \subset \mathcal{N}$, for all $x, \tilde{x}, y, \tilde{y} \in \mathcal{X}$, if $\mathcal{N}(x) = \mathcal{N}(y) = \mathcal{N}, \mathcal{N}(\tilde{x}) = \mathcal{N}(\tilde{y}) = \mathcal{M}$ and
$x_i = y_i$ for all $i \in \mathcal{N} \setminus \mathcal{M}$,
$x_j = \tilde{x}_j$ for all $j \in \mathcal{M}$,
$y_j = \tilde{y}_j$ for all $j \in \mathcal{M}$,
then $x \succeq y \iff \tilde{x} \succeq \tilde{y}$.

Social Expected Utility Hypothesis

-

Axiom: Social Expected Utility Hypothesis

There exists a continuous function $U : \mathcal{X} \to \mathbb{R}$ such that, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$:

$$\mathbf{x} \succeq \mathbf{y} \Longleftrightarrow \mathbb{E}\big(U(\mathbf{x})\big) \geq \mathbb{E}\big(U(\mathbf{y})\big)$$