# Social choice under uncertainty

## Beyond *ex ante* and *ex post*

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Université de Cergy-Pontoise
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- Social decisions invariably involve some degree of uncertainty.
- A fundamental principle of decision-making under uncertainty is *Statewise Dominance*: If policy $X$ leads to a better *ex post* outcome than policy $Y$ under any conceivable circumstances, then $X$ is better than $Y$, *ex ante*.
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# Introduction



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# Plan:

Part I. Harsanyi's Theorem and its discontents.

Part II. Spurious unanimity rears its ugly head.

Part III. Beyond *ex ante* and *ex post*.

# Part I

# Harsanyi Theorem and its discontents

# Basic notation

- Let $I$ be a finite set of individuals.
- Let $J$ be a finite set of possible states of nature.
  (Assume $|I| \geq 2$ and $|J| \geq 2$.)
- Let $\mathbf{X} = [x_j^i]_{j \in J}^{i \in I}$ denote an $I \times J$ real-valued matrix.
  ($I$=rows, $J$=columns.)
- For all $i \in I$ and $j \in J$, let $x_j^i$ represent the utility or consumption level of individual $i$ if state $j$ occurs.
- Thus, $\mathbf{X}$ represents a social prospect, which assigns a distinct payoff to each individual in each possible state of nature.
- Let $\mathcal{X} \subset \mathbb{R}^{I \times J}$ be the set of feasible social prospects.
  For simplicity, we will assume $\mathcal{X}$ is an open box in $\mathbb{R}^{I \times J}$.
- Let $\succeq$ represent an *ex ante* social welfare order on $\mathcal{X}$, perhaps representing the ethical judgements of a social observer.
- **Question:** What properties should $\succeq$ satisfy?

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# *Ex ante* Pareto



Given a social prospect $\mathbf{X} \in \mathbb{R}^{I \times J}$, we can write $\mathbf{X}$ as an $I$-indexed collection of $J$-dimensional "row vectors". For instance, if $I = \{1, 2, \ldots, n\}$, then

$$\mathbf{X} \quad = \quad \begin{bmatrix} \leftarrow & \mathbf{x}^1 & \rightarrow \\ \leftarrow & \mathbf{x}^2 & \rightarrow \\ & \vdots & \\ \leftarrow & \mathbf{x}^n & \rightarrow \end{bmatrix}, \quad \text{where } \mathbf{x}^1, \ldots, \mathbf{x}^n \in \mathbb{R}^J.$$

For each $i \in I$, row $\mathbf{x}^i$ is the *individual prospect* which $\mathbf{X}$ induces for $i$.

Let $\mathcal{X}^i := \{\mathbf{x}^i;\ \mathbf{X} \in \mathcal{X}\} \subset \mathbb{R}^J$ (the $i$th "row space").

Person $i$'s *ex ante* preferences are represented by a preorder $\succeq^i$ on $\mathcal{X}^i$

We will require the *ex ante* SWO $\succeq$ to satisfy the following axiom:

*Ex ante* PARETO: For all $i \in I$ and all $\mathbf{X}, \mathbf{Y} \in \mathcal{X}$, if $\mathbf{x}^h = \mathbf{y}^h$ for all $h \in I \setminus \{i\}$, then $\mathbf{X} \succeq \mathbf{Y}$ if and only if $\mathbf{x}^i \succeq^i \mathbf{y}^i$.

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Given a social prospect $\mathbf{X} \in \mathbb{R}^{I \times J}$, we can also write $\mathbf{X}$ as an $J$-indexed collection of $I$-dimensional "column vectors". If $J = \{1, \ldots, m\}$, then

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We will assume that the same outcomes are feasible in every state of nature:

*Identical Column Spaces*:
There is a single subset $\mathcal{X}_{\rm xp} \subset \mathbb{R}^I$ such that $\mathcal{X}_j = \mathcal{X}_{\rm xp}$ for all $j \in J$.

A (state-independent) *ex post* SWO is a preorder $\succeq_{\rm xp}$ on $\mathcal{X}_{\rm xp}$.
We do *not* require the social observer to be an SEU maximizer.
We only require the observer to satisfy a weak rationality condition....

SOCIAL STATEWISE DOMINANCE: For all $\mathbf{X}$, $\mathbf{Y} \in \mathcal{X}$, and all $j \in J$, if $\mathbf{x}_k = \mathbf{y}_k$ for all $k \in J \setminus \{j\}$, then $\mathbf{X} \succeq \mathbf{Y}$ if and only if $\mathbf{x}_j \succeq_{\rm xp} \mathbf{y}_j$.

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# Social Statewise Dominance



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$$\mathbf{X} \quad = \quad \begin{bmatrix} \uparrow & \uparrow & \cdots & \uparrow \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_m \\ \downarrow & \downarrow & \cdots & \downarrow \end{bmatrix}, \quad \text{where } \mathbf{x}_1, \ldots, \mathbf{x}_m \in \mathbb{R}^I$$

**Idea:** $\mathbf{x}_j$ = (*ex post* social outcome that $\mathbf{X}$ produces if state $j$ occurs).
Let $\mathcal{X}_j := \{\mathbf{x}_j;\ \mathbf{X} \in \mathcal{X}\} = \{$*ex post* social outcomes feasible in state $j\}$

We will assume that the same outcomes are feasible in every state of nature:

*Identical Column Spaces*:
There is a single subset $\mathcal{X}_{\mathrm{xp}} \subset \mathbb{R}^I$ such that $\mathcal{X}_j = \mathcal{X}_{\mathrm{xp}}$ for all $j \in J$.

A (state-independent) *ex post* SWO is a preorder $\succeq_{\mathrm{xp}}$ on $\mathcal{X}_{\mathrm{xp}}$.
We do *not* require the social observer to be an SEU maximizer.
We only require the observer to satisfy a weak rationality condition....

SOCIAL STATEWISE DOMINANCE: For all $\mathbf{X}$, $\mathbf{Y} \in \mathcal{X}$, and all $j \in J$, if $\mathbf{x}_k = \mathbf{y}_k$ for all $k \in J \setminus \{j\}$, then $\mathbf{X} \succeq \mathbf{Y}$ if and only if $\mathbf{x}_j \succeq_{\mathrm{xp}} \mathbf{y}_j$.

We also do *not* assume individuals are SEU maximizers.

We only assume that each individual satisfies a basic rationality condition:

INDIVIDUAL STATEWISE DOMINANCE: For all $i \in I$ and $j \in J$, and all $\mathbf{x}, \mathbf{y} \in \mathcal{X}^i$ with $x_k = y_k$ for all $k \in J \setminus \{j\}$, we have $\mathbf{x} \succeq^i \mathbf{y} \iff x_j \geq y_j$.

In fact, even this axiom is sort of optional. Instead, we could assume that the *ex post* social preference order $\succeq_{\text{xp}}$ satisfies:

*Ex post* PARETO: For all $i \in I$ and all $\mathbf{x}, \mathbf{y} \in \mathcal{X}_{\text{xp}}$ with $x^h = y^h$ for all $h \in I \setminus \{i\}$, we have $\mathbf{x} \succeq_{\text{xp}} \mathbf{y} \iff x^i \geq y^i$.

Our last axiom is a standard technical condition....

CONTINUITY: The order $\succeq$ is *continuous*, i.e., its upper and lower contour sets are closed subsets of $\mathcal{X}$. (Recall $\mathcal{X} \subseteq \mathbb{R}^{I \times J}$.)

# Individual statewise dominance and *Ex post Pareto*



We also do *not* assume individuals are SEU maximizers.

We only assume that each individual satisfies a basic rationality condition:

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**Theorem 1.** *Suppose $\mathcal{X}$ and $\succeq$ satisfy the above conditions. Then:*

(a) *Each individual $i \in I$ has an increasing, continuous* ex post *utility function $u^i : \mathcal{X}^i_{\rm xp} \longrightarrow \mathbb{R}$, such that $\succeq_{\rm xp}$ is represented by the utilitarian* ex post *social welfare function $W_{\rm xp} : \mathcal{X}_{\rm xp} \longrightarrow \mathbb{R}$ defined by*

$$W_{\rm xp}(\mathbf{x}) \quad := \quad \sum_{i \in I} u^i(x^i), \qquad \text{for all } \mathbf{x} \in \mathcal{X}_{\rm xp}.$$

(Here, for all $i \in I$, define $\mathcal{X}^i_{\rm xp} := \{x^i; \mathbf{x} \in \mathcal{X}_{\rm xp}\}$, an open interval in $\mathbb{R}$.)

(b) *There is a positive probability vector $\mathbf{p} \in \Delta_J$, such that, for all $i \in I$, the order $\succeq^i$ has an SEU representation $U^i_{\rm xa} : \mathcal{X}^i \longrightarrow \mathbb{R}$ given by*

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$$W(\mathbf{X}) \quad := \quad \overbrace{\sum_{j \in J} p_j \, W_{\rm xp}(\mathbf{x}_j)}^{\text{SEU representation w.r.t. (a)}} \quad = \quad \overbrace{\sum_{i \in I} U^i_{\rm xa}(\mathbf{x}^i)}^{\text{Utilitarian SWF w.r.t. (b)}}, \quad \text{for all } \mathbf{X} \in \mathcal{X}.$$

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## Interpretation.

This is similar to Harsanyi's (1955) *Social Aggregation Theorem*, but with two key differences:

- Harsanyi *assumes* all agents (i.e. all individuals and the planner) are expected utility maximizers (with vNM preferences on lotteries). In contrast, we *derive* an SEU representation for all the agents, from much weaker "monotonicity" axioms.
- Harsanyi assumes agents have von Neumann-Morgenstern (vNM) preferences over lotteries with *objective* probabilities. In contrast, we use a Savage-type framework: agents have preferences over "prospects". It turns out these preferences are determined by maximizing expected utility with respect to a *subjective* probability $\mathbf{p}$.

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Then, unless very implausible doxastic unanimity obtains, *ex ante* Pareto optimality and social rationality are *incompatible*.

**Theorem 1.** *Suppose $\mathcal{X}$ and $\succeq$ satisfy the above conditions. Then:*

**(a)** *For all $i \in I$, there is an increasing, continuous utility function $u^i : \mathcal{X}^i_{\rm xp} \longrightarrow \mathbb{R}$, such that $\succeq_{\rm xp}$ is represented by XP SWF $W_{\rm xp}(\mathbf{x}) := \sum_{i \in I} u^i(x^i)$, for all $\mathbf{x} \in \mathcal{X}_{\rm xp}$.*

**(b)** *There is a positive probability vector $\mathbf{p} \in \Delta_J$, such that for all $i \in I$, the order $\succeq^i$ has SEU representation $U^i_{\rm xa}(\mathbf{x}) := \sum_{j \in J} p_j\, u^i(x_j)$, for all $\mathbf{x} \in \mathcal{X}^i$.*

**(c)** *$\succeq$ is represented by $W(\mathbf{X}) := \sum_{j \in J} p_j\, W_{\rm xp}(\mathbf{x}_j) = \sum_{i \in I} U^i_{\rm xa}(\mathbf{x}^i)$, for all $\mathbf{X} \in \mathcal{X}$.*

**(d)** *$\mathbf{p}$ is unique, and $\{u^i\}_{i \in I}$ are unique up to common positive affine transformation.*

## Interpretation.

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**Earlier work:** Raiffa('68), Hylland&Zeckhauser('79), Hammond('81,'83), Seidenfeld et al. ('89, '91), Mongin('95,'98), Blackorby, Donaldson & Mongin ('04), Gilboa, Samet & Schmeidler ('04), Gajdos & Maurin ('04), Chambers & Hayashi ('06), Gajdos, Tallon & Vergnaud('08), Zuber('09), Fleurbaey('09,'11), Crès, Gilboa & Vieille('11), Keeney&Nau('11), Nascimento ('13),...

**Recent work:** Gilboa, Samuelson & Schmeidler ('14), Gayer, Gilboa, Samuelson & Schmeidler ('14), Alon & Gayer ('14), Danan, Gajdos, Hill&Tallon('14), Billot&Vergopoulos ('14), Qu('14)...

# Part II

# Spurious unanimity rears its ugly head

# Spurious Unanimity 

St.wise Dom. seems non-negotiable. Is Ex ante Pareto the culprit?

Indeed, Ex ante Pareto is already suspect, for other reasons.

To see this, suppose $J = \{h, t\}$ and $I = \{\text{Ann, Bob}\}$, with the beliefs:

| | $h$ | $t$ |
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| Ann's probability | 0.9 | 0.1 |
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(i.e. $p_{\text{Ann}}(h) = 0.9$, etc.)

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$\mathbf{X} \succ_A \mathbf{Y}$, because $\mathbb{E}(\mathbf{X}|u_A, p_A) = 7 > 0 = \mathbb{E}(\mathbf{Y}|u_A, p_A)$. Likewise, $\mathbf{X} \succ_B \mathbf{Y}$.

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**Theorem.** (GSS, 2004) *Let $W$ be an* ex post *social welfare function on $\mathcal{A}$, let $P$ be a probability distribution on $\mathcal{J}$, and let $\succeq$ be the* ex ante *social preference relation on $\mathcal{A}^{\mathcal{J}}$ which maximizes the $P$-expected value of $W$. Then $\succeq$ satisfies the* ex ante *Pareto condition restricted to admissible prospects if and only if $W$ is a weighted utilitarian sum of the individual utilities $u_i$, and $P$ is a weighted average of the individual probabilities $p_i$.*

# Gilboa, Samet & Schmeidler: restricted *ex ante* Pareto 

**Theorem.** (GSS'04) *Let $W$ be an* ex post *SWF on $\mathcal{A}$, let $P$ be a probability on $\mathcal{J}$, and let $\succeq$ be the* ex ante *preference relation on $\mathcal{A}^{\mathcal{J}}$ which maximizes the $P$-expected value of $W$. Then $\succeq$ satisfies the restricted* ex ante *Pareto condition $\iff$ $W$ is a weighted utilitarian sum of the utilities $\{u_i\}$, and $P$ is a weighted average of the probabilities $\{p_i\}$.*

This seems like a perfect solution. It does *not* require probability agreement, and it is *not* susceptible to spurious unanimity. ........ Or is it?

Suppose $\mathcal{J} = \{r, s, t\}$ and $I = \{\text{Ann, Bob}\}$.

Consider two prospects, $f$ and $g$, which yield the *same* payoff for both agents in each state of nature.

| | $r$ | $s$ | $t$ |
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| $f$ | 100 | 0 | 100 |
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Ann and Bob begin with the *same* prior probability $p$:

$$p(r) = 0.49, \quad p(s) = 0.02, \quad \text{and} \quad p(t) = 0.49.$$

Ann privately observes the event $\{r, s\}$, while Bob privately observes $\{s, t\}$.

After Bayesian updating, they have the following posterior probabilities:

| | Info | $r$ | $s$ | $t$ |
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Thus, $f \succ_{\mathrm{Ann}} g$ and $f \succ_{\mathrm{Bob}} g$.

Furthermore, $\mathfrak{B} = \{\mathcal{J}, \{r, t\}, \{s\}, \emptyset\}$, so both $f$ and $g$ are admissible.

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Indeed, if $P$ is the average of Ann's and Bob's beliefs (as GSS recommend), then $P$ also says Expected SWF($f$) $= 96$, while Expected SWF($g$) $= 4$.

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# Objective vs. subjective uncertainty



GSS attempt to distinguish between "legitimate" unanimity and "spurious" unanimity by an *endogenous* criterion: the agreement set $\mathfrak{B}$.

But this attempt fails. Maybe instead we should use an *exogenous* criterion.

**Idea:** We should distinguish between *objective* randomness (i.e. "risk") and *subjective* randomness (arising from "uncertainty").

- *Ex ante* Pareto only makes sense for *objective* randomness, where the agents can agree for legitimate reasons.
- *Ex ante* Pareto is never appropriate for *subjective* randomness, where "spurious unanimity" is possible.

Instead, we consider a model of social choice which with two independent sources of randomness: one objective and one subjective.

We apply *ex ante* Pareto *only* to the agents' preferences over *objective* randomness. This yields a new version of the Social Aggregation Theorem:

- *Ex ante* social preferences maximize expected value of a utilitarian SWF.
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# Part III

# Beyond *ex ante* and *ex post*

We will now use *three* indexing sets:

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# Social choice with twofold uncertainty: setup 2



Recall: $I = \{\text{individuals}\}$, and $J \times K = \{\text{states of nature}\}$.

- For all $i \in I$, $\mathbb{R}^{J \times K} = \{\text{personal prospects for individual } i\}$.
- Thus, for all $i \in I$ and $j \in J$,
  $\mathbb{R}^K = \{$personal prospects for $i$, *conditional* on observing event $j\}$.
- Likewise, for all $j \in J$,
  $\mathbb{R}^{I \times K} = \{$social prospects, *conditional* on observing event $j\}$.
- Finally, $\mathbb{R}^I = \{$*ex post* social outcomes$\}$.
  (The same social outcomes are feasible in every state of nature.)

We will interpret $J$ and $K$ as two "independent" sources of uncertainty.
Let $K$ be a source of "objective" uncertainty (e.g. a "roulette lottery").
Thus, it is reasonable to suppose that agents form probabilistic beliefs about $K$ —perhaps even the *same* probabilistic beliefs.

Let $J$ be a source of "subjective" uncertainty (e.g. a "horse lottery").
Thus, agents might *not* form probabilistic beliefs about $J$.
Even if they do, these beliefs may reasonably disagree....

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# Ex post Pareto and Coordinate Monotonicity



Let $\succeq$ be the *ex ante* social welfare order on $\mathbb{R}^{I\times J\times K}$.

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A basic rationality condition would then be:

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We will strengthen these axioms to INVARIANT $J$-PREFERENCES and INVARIANT $K$-PREFERENCES, by requiring the conditional social preference orders $\succeq_j$ and ${}_k\succeq$ to be independent of $j$ and $k$.

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We will condition the *ex ante* social preferences on *partial* information....

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# Pareto axioms



For all $i \in I$, recall that $\mathbb{R}^{J \times K}$ is the space of individual prospects for $i$.

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EX ANTE PARETO: For all $i \in I$, and any $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{I \times J \times K}$ with $\mathbf{x}^{i'} = \mathbf{y}^{i'}$ for all $i' \in I \setminus \{i\}$, we have $\mathbf{X} \succeq \mathbf{Y} \iff \mathbf{x}^i \succeq^i \mathbf{y}^i$.

But when agents have different subjective probabilities, there is a possibility for *spurious unanimity*. Then EX ANTE PARETO is very problematic.

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**Theorem 2.** *The ex ante SWF* $\succeq$ *satisfies* CONTINUITY, COORD MONO, INVARIANT $J$-PREFS, INVARIANT $K$-PREFS, *and* $J$-CONDITIONAL EX ANTE PARETO *if and only if the following holds:*

**(a)** $\succeq$ *satisfies* SOCIAL STATEWISE DOMINANCE. *For all* $i \in I$, *there is a continuous, increasing utility function* $u^i : \mathbb{R} \longrightarrow \mathbb{R}$ *such that the ex post social welfare order* $\succeq_{\rm xp}$ *is represented by the utilitarian SWF*

$$W_{\rm xp}(\mathbf{x}) := \sum_{i \in I} u^i(x^i), \qquad \text{for all } \mathbf{x} \in \mathbb{R}^I.$$

**(b)** *There exists* $\mathbf{q} \in \Delta_K$ *such that, for all* $i \in I$, *the order* $\succeq^i_J$ *is has an SEU representation given by the function* $U^i(\mathbf{x}) := \sum_{k \in K} {}_kq\, u^i({}_kx)$, *for all* $\mathbf{x} \in \mathbb{R}^K$.

**(c)** $\succeq_J$ *has a utilitarian SWF:* $W_J(\mathbf{x}) := \sum_{i \in I} U^i(\mathbf{x}^i)$, *for all* $\mathbf{x} \in \mathbb{R}^{I \times K}$.

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- Theorem 2 salvages "social rationality" (an SEU representation for $\succeq$) by weakening (but not eliminating) the *ex ante* Pareto axiom.
- The social preferences $\succeq$ are still bound by unanimous preferences over "objective" social prospects (i.e. those depending only on $K$).
- This implies *ex post* utilitarianism, as well as a "conditional" form of *ex ante* utilitarianism.
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- In the ranking of social alternatives, three simple monotonicity axioms (Ex Ante Pareto, Social Statewise Dominance, and Ex Post Pareto/Individual Statewise Dominance) yield a generalization of Harsanyi's Social Aggregation Theorem (Theorem 1).
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- By introducing a model of social choice with *twofold* uncertainty, we resolve this paradox, and suggest a possible reconciliation between social rationality and the Pareto axiom (Theorem 2).

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## Theorem 2

### Statement

### Remarks

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