# Aggregating Tastes, Beliefs, and Attitudes under Uncertainty

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# Motivation

Social decisions in uncertain environments where individual agents have both different tastes/interests and different beliefs

SEU preferences:

- Tastes captured by a utility function
- Beliefs captured by a prior

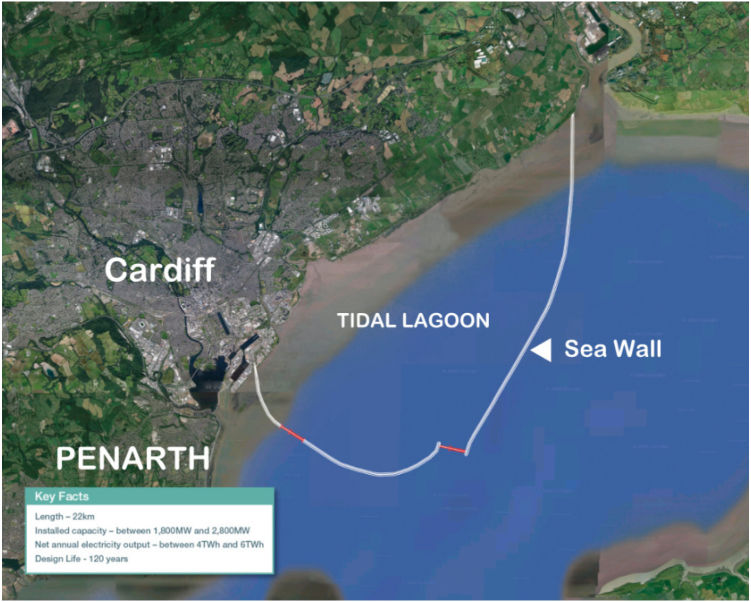
Paretian aggregation impossible unless common beliefs

Non-SEU (ambiguity) preferences:

- Tastes captured by a utility function
- Beliefs captured by (e.g.) a set of priors
- Ambiguity attitudes

Paretian aggregation impossible even when common beliefs

# Example


Cardiff
TIDAL LAGOON
Sea Wall
PENARTH
Key Facts
Length – 22km
Installed capacity – between 1,800MW and 2,800MW
Net annual electricity output – between 4TWh and 6TWh
Design Life - 120 years

# Example

| | $\Delta t^\circ < 2$ | | $\Delta t^\circ > 2$ | |
|---|---|---|---|---|
| Nothing | ♓♓♓ | $\$$ | | $\$$ |
| Treatment | ♓♓♓ | $\$$ | ♓ | $\$\$\$$ |
| Prevention | ♓♓ | $\$\$$ | ♓♓ | $\$\$$ |

SEU preferences:

- Fishery sector believes $\Delta t^\circ = 3$
- Tech sector believes $\Delta t^\circ = 1$

Spurious unanimity

Non-SEU preferences:

- Both sectors believe $1 \leq \Delta t^\circ \leq 3$
- Both sectors are ambiguity averse (MEU)

"Spurious hedging"

# Contribution

Introduce a weakening of Pareto Domiannce – Unambiguous Pareto Dominance – that is immune to spurious hedging
(and a further weakening of this axiom that is immune to spurious unanimity as well)

Show that these axioms restore the possibility of preference aggregation for ambiguity sensitive decision makers
and relate social beliefs to individual beliefs (independently of ambiguity attitudes)

Within a general class of preferences and a flexible notion of beliefs so results applicable to many popular models of decision under uncertainty

Introduce a new axiom allowing aggregation of ambiguity attitudes, independently of beliefs and tastes

# Related literature

Paretian aggregation of SEU preferences:

- Hylland, Zeckhauser (1979); Mongin (1995, 1997, 1998); Chambers, Hayashi (2014); Billot, Vergopoulos (2014)

Paretian aggregation of ambiguity preferences:

- Gajdos, Tallon, Vergnaud (2008); Crès, Gilboa, Vieille (2011); Nascimento (2012); Herzberg (2013); Qu (2014)

Weakenings of Pareto Dominance:

- Gilboa, Samet, Schmeidler (2004); Gilboa, Samuelson, Schmeidler (2014); Gayer, Gilboa, Samuelson, Schmeidler (2014); Alon, Gayer (2014); Blume, Cogley, Easley, Sargent, Tsyrennikov (2014); Brunnermeier, Simsek, Xiong (2014)

# Outline

# Anscombe-Aumann acts

$\mathcal{S}$ a finite ste of states

$\mathcal{X}$ a finite set of prizes

$\mathcal{P} = \Delta(\mathcal{X})$ the set of lotteries

$\mathcal{F} = \mathcal{P}^{\mathcal{S}}$ the set of acts

- $\lambda f + (1 - \lambda) g$ statewise mixing
- $\mathcal{P} \subset \mathcal{F}$ constant acts
- $u \circ f = (E_{f(s)}(u(x)))_{s \in \mathcal{S}}$ utility act

$\succsim \subseteq \mathcal{F} \times \mathcal{F}$ a preference relation

- $\succ$ and $\sim$ the asymmetric and symmetric parts of $\succsim$

# SEU preference relation

## Axiom (Completeness)

For all $f, g \in \mathcal{F}$, $f \succsim g$ or $g \succsim f$.

## Axiom (Transitivity)

For all $f, g, h \in \mathcal{F}$, if $f \succsim g$ and $g \succsim h$ then $f \succsim h$.

## Axiom (Non-Triviality)

There exist $f, g \in \mathcal{F}$ such that $f \succ g$.

## Axiom (Monotonicity)

For all $f, g \in \mathcal{F}$, if $f(s) \succsim g(s)$ for all $s \in \mathcal{S}$ then $f \succsim g$.

## Axiom (Mixture Continuity)

For all $f, g, h \in \mathcal{F}$, the sets $\{\lambda \in [0,1] : \lambda f + (1-\lambda)g \succsim h\}$ and $\{\lambda \in [0,1] : h \succsim \lambda f + (1 - \lambda g)\}$ are closed.

## Axiom (Independence)

For all $f, g, h \in \mathcal{F}$ and $\lambda \in (0,1)$, if $f \succsim g$ then $\lambda f + (1-\lambda)h \succsim \lambda g + (1-\lambda)h$.

# SEU representation

## Proposition (Anscombe, Aumann, 1963)

*A binary relation $\succsim$ on $\mathcal{F}$ satisfies Completeness, Transitivity, Non-Triviality, Monotonicity, Mixture Continuity, Independence if and only if there exist a non-constant function $u : \mathcal{X} \to \mathbb{R}$ and a probability distribution $m \in \Delta(\mathcal{S})$ such that for all $f, g \in \mathcal{F}$,*

$$f \succsim g \Leftrightarrow E_m(u \circ f) \geq E_m(u \circ g).$$

*(Moreover, $u$ is unique up to a positive affine transformation, and $m$ is unique.)*

$m$ is the agent's prior.

# Outline

Setup

Preference and unambiguous preference

Pareto Dominance

Unambiguous Pareto Dominance

Applications
- MEU preferences
- CEU preferences
- Smooth Ambiguity preferences

Aggregating ambiguity attitudes

# MBA preference relation

Weaken Independence to:

**Axiom (Risk Independence)**

For all $p, q, r \in \mathcal{P}$ and $\lambda \in (0,1)$, if $p \succsim q$ then $\lambda p + (1-\lambda) r \succsim \lambda q + (1-\lambda) r$.

# MBA representation

## Proposition (Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci, Siniscalchi, 2011)

*A binary relation* $\succsim$ *on* $\mathcal{F}$ *satisfies Completeness, Transitivity, Non-Triviality, Monotonicity, Mixture Continuity, Risk Independence if and only if*
*there exist a non-constant function* $u : \mathcal{X} \to \mathbb{R}$ *and a monotonic, continuous, normalized functional* $J : \mathrm{conv}(u(\mathcal{X}))^{\mathcal{S}} \to \mathbb{R}$ *such that for all* $f, g \in \mathcal{F}$,

$$f \succsim g \Leftrightarrow J(u \circ f) \geq J(u \circ g).$$

*(Moreover,* $u$ *is unique up to a positive affine transformation, and* $J$ *is unique given* $u$*.)*

# Unambiguous preference relation

$\succsim^*$, subrelation of $\succsim$:
for all $f, g \in \mathcal{F}$, if $f \succsim^* g$ then $f \succsim g$.

# Bewley preference relation

Weaken completeness to:

**Axiom (Risk Completeness)**

For all $p, q \in \mathcal{P}$, $p \succsim q$ or $q \succsim p$.

But maintain Independence.

# Bewley representation

## Proposition (Bewley, 2002)

*A binary relation $\succsim^*$ on $\mathcal{F}$ satisfies Risk Completeness, Transitivity, Non-Triviality, Monotonicity, Mixture Continuity, Independence if and only if there exist a non-constant function $u : \mathcal{X} \to \mathbb{R}$ and a non-empty, compact, convex set $M \subseteq \Delta(\mathcal{S})$ of probability distributions such that for all $f, g \in \mathcal{F}$,*

$$f \succsim^* g \Leftrightarrow [E_m(u \circ f) \geq E_m(u \circ g) \text{ for all } m \in M] \,.$$

*(Moreover, $u$ is unique up to a positive affine transformation, and $M$ is unique.)*

$M$ is the agent's set of relevant priors.

# Examples

**Example (Ghirardato, Maccheroni, Marinacci, 2004; Nehring, 2007)**

Define $f \succsim^{*\mathsf{GMMN}} g$ if and only if
$\lambda f + (1-\lambda)h \succsim \lambda g + (1-\lambda)h$ for all $h \in \mathcal{F}$ and $\lambda \in (0,1]$.
Define $M^{\mathsf{GMMN}}$ the corresponding set of relevant priors.

**Example (Klibanoff, Mukerji, Seo, 2014)**

Consider acts on $\mathcal{S}^\infty$ (iid).
Identify a non-empty, closed set $R \subseteq \Delta(\mathcal{S})$ of "relevant measures" through bets on limiting frequency over $\mathcal{S}$.
Define $M^{\mathsf{KMS}} = \mathrm{conv}(R)$.
Define $\succsim^{*\mathsf{KMS}}$ the corresponding unambiguous preference relation.

# Generalized Hurwicz representation

## Proposition (CGMMS, 2011)

*$\succsim$ is an MBA preference relation on $\mathcal{F}$ with unambiguous part $\succsim^*$ if and only if there exist a non-constant function $u : \mathcal{X} \to \mathbb{R}$, a non-empty, compact, convex set $M \subseteq \Delta(\mathcal{S})$ and a function $\alpha : \mathcal{F} \to [0,1]$ such that:*

- *$(u, M)$ is a Bewley representation of $\succsim^*$,*
- *The functional $J : \mathrm{conv}(u(\mathcal{X}))^{\mathcal{S}} \to \mathbb{R}$ defined by, for all $f \in \mathcal{F}$,*

$$J(u \circ f) = \alpha(f) \min_{m \in M} E_m(u \circ f) + (1 - \alpha(f)) \max_{m \in M} E_m(u \circ f)$$

  *is monotonic, continuous, and normalized, and $(u, J)$ is an MBA representation of $\succsim$.*

*(Moreover, $u$ is unique up to a positive affine transformation, $M$ is unique, and $\alpha$ is unique on $\succsim^*$-non-crisp acts.)*

# Outline

# Setup

$\mathcal{I}$ a finite set of individuals

$\mathcal{I}' = \{0\} \cup \mathcal{I}$, 0 for society

$\succsim_i \subseteq \mathcal{F} \times \mathcal{F}$ a preference relation, for all $i \in \mathcal{I}'$

## Definition

Individual $i \in \mathcal{I}$ is null if there exist no $p, q \in \mathcal{P}$ such that $p \succ_0 q$ and $p \sim_j q$ for all $j \in \mathcal{I} \setminus \{i\}$.

# Axioms

### Axiom (Risk Diversity)

For all $i \in \mathcal{I}$, there exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ and $p \sim_j q$ for all $j \in \mathcal{I} \setminus \{i\}$.

### Axiom (Risk Minimal Agreement)

There exist $p, q \in \mathcal{P}$ such that $p \succ_i q$ for all $i \in \mathcal{I}$.

### Axiom (Pareto Dominance)

For all $f, g \in \mathcal{F}$, if $f \succsim_i g$ for all $i \in \mathcal{I}$ then $f \succsim_0 g$.

### Axiom (Risk Pareto Dominance)

For all $p, q \in \mathcal{P}$, if $p \succsim_i q$ for all $i \in \mathcal{I}$ then $p \succsim_0 q$.

# Risk Pareto Dominance

## Proposition (Harsanyi, 1955)

*Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ for all $i \in \mathcal{I}'$.*
*Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Risk Pareto Dominance*
*if and only if*
*for all MBA representations $(u_i, J_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ and $\gamma \in \mathbb{R}$ such that*

$$u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma.$$

*Moreover, if $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity then $\theta$ and $\gamma$ are unique given $(u_i)_{i\in\mathcal{I}'}$ and an individual $i \in \mathcal{I}$ is null if and only if $\theta_i = 0$.*

# Spurious unanimity

**Example**

$\mathcal{S} = \{s_1, s_2\}$, $\mathcal{X} = \{x, y, z\}$, $\mathcal{I} = \{1, 2\}$.
$\succsim_i$ is SEU with representation $(u_i, m_i)$ for all $i = 0, 1, 2$, where

$$\begin{aligned} u_1(x) &= 1, u_1(y) = 0, u_1(z) = 0, \qquad && m_1(s_1) = \tfrac{1}{4}, \\ u_2(x) &= 0, u_2(y) = 1, u_2(z) = 0, \qquad && m_2(s_1) = \tfrac{3}{4}. \end{aligned}$$

Then $(\succsim_i)_{i=0,1,2}$ can only satisfy Pareto Dominance if 1 or 2 is null.
Indeed, suppose $\theta_1\theta_2 > 0$ (w.l.o.g. $\theta_1 + \theta_2 = 1$), and define

$$\begin{aligned} f(s_1) &= p, f(s_2) = q, & & & p &= \theta_2 x + \theta_1 z, \\ & & \text{where} & & & \\ g(s_1) &= q, g(s_2) = p, & & & q &= \theta_1 y + \theta_2 z. \end{aligned}$$

Then $g \succ_i f$ for $i = 1, 2$ but $f \sim_0 g$.

# Spurious hedging

## Example

$\mathcal{S} = \{s_1, s_2\}$, $\mathcal{X} = \{x, y, z\}$, $\mathcal{I} = \{1, 2\}$.
$\succsim_i$ is MEU with representation $(u_i, M_i)$ for all $i = 0, 1, 2$, where

$$\begin{aligned} u_1(x) = 1, u_1(y) = 0, u_1(z) = 0, \\ u_2(x) = 0, u_2(y) = 1, u_2(z) = 0, \end{aligned} \qquad M_1 = M_2 = \left[\tfrac{1}{4}, \tfrac{3}{4}\right].$$

Then $(\succsim_i)_{i=0,1,2}$ can only satisfy Pareto Dominance if 1 or 2 is null.
Indeed, suppose $\theta_1\theta_2 > 0$ (w.l.o.g. $\theta_1 + \theta_2 = 1$), and define

$$\begin{aligned} f(s_1) = p, f(s_2) = q, \\ g(s_1) = q, g(s_2) = p, \end{aligned} \qquad \text{where} \qquad \begin{aligned} p = \theta_2 x + \theta_1 z, \\ q = \theta_1 y + \theta_2 z. \end{aligned}$$

Then $\frac{1}{2}f + \frac{1}{2}g \succ_i f \sim_i g$ for $i = 1, 2$ but $\frac{1}{2}f + \frac{1}{2}g \sim_0 f \sim_0 g$.

# Outline

Setup

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# Unambiguous Pareto Dominance

## Axiom (Unambiguous Pareto Dominance)

For all $f, g \in \mathcal{F}$, if $f \succsim_i^* g$ for all $i \in \mathcal{I}$ then $f \succsim_0^* g$.

# Unambiguous Pareto Dominance

## Theorem

*Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ with unambiguous part $\succsim_i^*$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity.*
*Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Unambiguous Pareto Dominance*
*if and only if*
*for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ and $\gamma \in \mathbb{R}$ such that $u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma$ and*

$$M_0 \subseteq \bigcap_{i\in\mathcal{I},\theta_i>0} M_i.$$

# Common-Taste Unambiguous Pareto Dominance

## Definition

$f$ and $g$ are common-taste acts if $p \succsim_i q \Leftrightarrow p \succsim_j q$ for all $i, j \in \mathcal{I}$ and $p, q \in \mathrm{conv}(f(\mathcal{S}) \cup g(\mathcal{S}))$

## Axiom (Common-Taste Unambiguous Pareto Dominance)

For all common-taste acts $f, g \in \mathcal{F}$, if $f \succsim_i^* g$ for all $i \in \mathcal{I}$ then $f \succsim_0^* g$.

# Common-Taste Unambiguous Pareto Dominance

## Theorem

*Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ with unambiguous part $\succsim_i^*$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Minimal Agreement.*
*Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Common-Taste Unambiguous Pareto Dominance*
*if and only if*
*for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ and $\gamma \in \mathbb{R}$ such that $u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma$ and*

$$M_0 \subseteq \operatorname{conv}\left(\bigcup_{i\in\mathcal{I}} M_i\right).$$

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Setup

Preference and unambiguous preference

Pareto Dominance

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Setup

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Pareto Dominance

Unambiguous Pareto Dominance

Applications
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Aggregating ambiguity attitudes

# MEU preferences

MEU preferences correspond to MBA functionals of the form

$$J(u \circ f) = \min_{m \in M} E_m(u \circ f),$$

where $M \subseteq \Delta(\mathcal{S})$ is non-empty, compact, convex. ($M$ is unique.)

We have:

$$M^{\mathsf{GMMN}} = M^{\mathsf{KMS}} = M.$$

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Setup

Preference and unambiguous preference

Pareto Dominance

Unambiguous Pareto Dominance

Applications

- MEU preferences
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# CEU preferences

CEU preferences correspond to MBA functionals of the form

$$J(u \circ f) = \int_{\mathcal{S}} u \circ f \; d\nu,$$

where $\nu : 2^{\mathcal{S}} \to [0,1]$ is a capacity and the integral is Choquet. ($\nu$ is unique.) Equivalently, writing $\mathcal{S} = \{s_1, \ldots, s_N\}$,

$$J(u \circ f) = E_{m_{\nu,\sigma}}(u \circ f)$$

where $u \circ f(s_{\sigma(1)}) \geq \ldots \geq u \circ f(s_{\sigma(N)})$ and

$$m_{\nu,\sigma}(s_n) = \nu(\{s_{\sigma(1)}, \ldots, s_{\sigma(n)}\}) - \nu(\{s_{\sigma(1)}, \ldots, s_{\sigma(n-1)}\})$$

We have:

$$M^{\text{GMMN}} = \text{conv}\left(\{m_{\nu,\sigma} : \sigma \in \text{perm}(N)\}\right).$$

# Unambiguous Pareto Dominance

## Corollary

*Assume that* $\succsim_i$ *is a CEU preference relation on* $\mathcal{F}$ *with unambiguous part* $\succsim_i^{*\mathrm{GMMN}}$ *for all* $i \in \mathcal{I}'$ *and that* $(\succsim_i)_{i\in\mathcal{I}}$ *satisfies Risk Diversity.*
*Then* $(\succsim_i)_{i\in\mathcal{I}'}$ *satisfies Unambiguous Pareto Dominance*
*if and only if*
*for all CEU representations* $(u_i, \nu_i)_{i\in\mathcal{I}'}$ *of* $(\succsim_i)_{i\in\mathcal{I}'}$, *there exist* $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ *and* $\gamma \in \mathbb{R}$ *such that* $u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma$ *and for all* $\sigma \in \mathrm{perm}(N)$,

$$m_{\nu_0,\sigma} \in \bigcap_{i\in\mathcal{I},\theta_i>0} \mathrm{conv}(\{m_{\nu_i,\tau} : \tau \in \mathrm{perm}(N)\}).$$

## Common-Taste Unambiguous Pareto Dominance

### Corollary

*Assume that* $\succsim_i$ *is a CEU preference relation on* $\mathcal{F}$ *with unambiguous part* $\succsim_i^{*\mathrm{GMMN}}$ *for all* $i \in \mathcal{I}'$ *and that* $(\succsim_i)_{i\in\mathcal{I}}$ *satisfies Risk Minimal Agreement.*
*Then* $(\succsim_i)_{i\in\mathcal{I}'}$ *satisfies Common-Taste Unambiguous Pareto Dominance*
*if and only if*
*for all CEU representations* $(u_i, \nu_i)_{i\in\mathcal{I}'}$ *of* $(\succsim_i)_{i\in\mathcal{I}'}$*, there exist* $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ *and* $\gamma \in \mathbb{R}$ *such that* $u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma$ *and for all* $\sigma \in \mathrm{perm}(N)$,

$$m_{\nu_0,\sigma} \in \mathrm{conv}(\{m_{\nu_i,\tau} : \tau \in \mathrm{perm}(N), i \in \mathcal{I}\}).$$

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Setup

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Unambiguous Pareto Dominance

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# Smooth Ambiguity preferences

Smooth Ambiguity preferences correspond to MBA functionals of the form

$$J(u \circ f) = \phi^{-1}(E_\mu(\phi(E_m(u \circ f)))),$$

where $\phi : \mathrm{conv}(u(\mathcal{X})) \to \mathbb{R}$ is continuous and strictly increasing and $\mu$ is a countably additive probability measure over $\Delta(\mathcal{S})$. ($\mu$ is unique and $\phi$ is unique up to a positive affine transformation given $u$.)

If $(u, \mu, \phi)$ is regular (e.g. if $\mathrm{supp}(\mu)$ is finite) then we have:

$$M^{\mathrm{KMS}} = \mathrm{conv}(\mathrm{supp}(\mu)).$$

# Unambiguous Pareto Dominance

## Corollary

*Assume that $\succsim_i$ is a regular SA preference relation on $\mathcal{F}$ with unambiguous part $\succsim_i^{*\mathsf{KMS}}$ for all $i \in \mathcal{I}'$ and that $(\succsim_i)_{i\in\mathcal{I}}$ satisfies Risk Diversity.*
*Then $(\succsim_i)_{i\in\mathcal{I}'}$ satisfies Unambiguous Pareto Dominance*
*if and only if*
*for all regular SA representations $(u_i, \mu_i, \phi_i)_{i\in\mathcal{I}'}$ of $(\succsim_i)_{i\in\mathcal{I}'}$, there exist $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ and $\gamma \in \mathbb{R}$ such that $u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma$ and*

$$\mathrm{supp}(\mu_0) \subseteq \bigcap_{i\in\mathcal{I},\theta_i>0} \mathrm{conv}(\mathrm{supp}(\mu_i)).$$

# Common-Taste Unambiguous Pareto Dominance

## Corollary

*Assume that* $\succsim_i$ *is a regular SA preference relation on* $\mathcal{F}$ *with unambiguous part* $\succsim_i^{*\mathsf{KMS}}$ *for all* $i \in \mathcal{I}'$ *and that* $(\succsim_i)_{i\in\mathcal{I}}$ *satisfies Risk Minimal Agreement.*
*Then* $(\succsim_i)_{i\in\mathcal{I}'}$ *satisfies Common-Taste Unambiguous Pareto Dominance*
*if and only if*
*for all regular SA representations* $(u_i, \mu_i, \phi_i)_{i\in\mathcal{I}'}$ *of* $(\succsim_i)_{i\in\mathcal{I}'}$*, there exist* $\theta \in \mathbb{R}_+^{\mathcal{I}} \setminus \{0\}$ *and* $\gamma \in \mathbb{R}$ *such that* $u_0 = \sum_{i\in\mathcal{I}} \theta_i u_i + \gamma$ *and*

$$\mathrm{supp}(\mu_0) \subseteq \mathrm{conv}\left(\bigcup_{i\in\mathcal{I}} \mathrm{supp}(\mu_i)\right).$$

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Setup

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Applications
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Aggregating ambiguity attitudes

# Uncertainty-Adjusted Pareto Dominance

## Definition

$\underline{p}$ is a lower certainty-equivalent of $f$ if for all $p \in \mathcal{P}$,

$$f \succsim^* p \Leftrightarrow \underline{p} \succsim p.$$

$\overline{p}$ is an upper certainty-equivalent of $f$ if for all $p \in \mathcal{P}$,

$$p \succsim^* f \Leftrightarrow p \succsim \overline{p}.$$

## Axiom (Uncertainy-Adjusted Pareto Dominance)

For all $f \in \mathcal{F}$, all lower and upper certainty equivalents $(\underline{p}_i, \overline{p}_i)_{i \in \mathcal{I}} \in (\mathcal{P}^2)^{\mathcal{I}'}$ of $f$, and all $\lambda \in [0, 1]$,

- if $f \succsim_i \lambda \underline{p}_i + (1 - \lambda)\overline{p}_i$ for all $i \in \mathcal{I}$ then $f \succsim_0 \lambda \underline{p}_0 + (1 - \lambda)\overline{p}_0$,
- if $\lambda \underline{p}_i + (1 - \lambda)\overline{p}_i \succsim_i f$ for all $i \in \mathcal{I}$ then $\lambda \underline{p}_0 + (1 - \lambda)\overline{p}_0 \succsim_0 f$.

# Uncertainty-Adjusted Pareto Dominance

**Theorem**

*Assume that $\succsim_i$ is an MBA preference relation on $\mathcal{F}$ with unambiguous part $\succsim_i^*$ for all $i \in \mathcal{I}'$.*
*Then $(\succsim_i)_{i \in \mathcal{I}'}$ satisfies Uncertainty-Adjusted Pareto Dominance if and only if for all generalized Hurwicz representations $(u_i, M_i, \alpha_i)_{i \in \mathcal{I}'}$ of $(\succsim_i)_{i \in \mathcal{I}'}$ and every $f \in \mathcal{F}$ that is not $\succsim_0$-crisp,*

$$\alpha_0(f) \in \mathrm{conv}(\{\alpha_i(f) : i \in \mathcal{I}, f \text{ is } \succsim_i\text{-non-crisp}\}).$$