Utilitarianism with Prior Heterogeneity

Antoine Billot [1] and Vassili Vergopoulos [2]

[1] LEMMA - Université Paris 2 Panthéon-Assas and IUF

[2] Paris School of Economics - Université Paris 1 Panthéon-Sorbonne

Social choice and welfare under uncertainty
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Introduction

Harsanyi (1955): if individuals and society are VNM expected utility maximizers, then the Pareto condition implies utilitarianism.

However, in a Savage framework where individuals might have heterogeneous beliefs:

- the same argument fails to justify utilitarianism,
- SEU at the social level and the Pareto condition become incompatible.

Mongin (1997) identifies an issue in blind applications of the Pareto condition: *spurious unanimity*. Should social preferences be constrained by unanimity when individuals are unanimous for radically different reasons?

Gilboa, Samet and Schmeidler (GSS, 2004) restrict the Pareto condition to nonspurious unanimity and derive the utilitarian rule.

GSS's duel example

Two gentlemen, G1 and G2, contemplate the possibility of fighting in a duel.

	G1 wins	G2 wins
λ_1	.85	.15
λ_2	.15	.85
no duel	$(0,0)$	$(0,0)$
duel	$(1,-5)$	$(-5,1)$

The Pareto condition supports the duel.
Subjective expected utility maximization rejects the duel.

The father example

A father of two children must make a choice between funding two BAs or one PhD only. In the latter case, it is the next school test that will determine the one child who gets the PhD.

	Alice	Bob
λ_A	.33	.66
λ_B	.66	.33
BA	$(3,3)$	$(3,3)$
PhD	$(8,0)$	$(0,8)$

The Pareto condition supports the BA option.
Subjective expected utility maximization supports the PhD option.

In the duel example, not everyone can win. Here, everyone getting a BA is feasible. Even if unanimity is spurious here too, do we really want to reject it?

This paper

It is not clear why the state space of society should be the same one as the state space of individuals.

A *social state* (or *state of opinion*) is here rather defined as an element of $\Omega = S^N$, where S is the state space of individuals and N is the set of individuals.

At state $\omega \in \Omega$, individual $i \in N$ thinks that the true state is $\omega_i \in S$.

Retrospectively, the assumption $\Omega = S$ means that individuals always form the same opinion on the true state in S.

Aggregation rule:

- Social probability is the independent product of individual probabilities,
- Social utility is a convex combination of individual utilities. Hence utilitarianism.

The basic tension between the Pareto condition and SEU disappears.
Actually, the Pareto condition and SEU still produce an axiomatic justification of utilitarianism as in Harsanyi's theorem.

This presentation

The Diamond (1967) example

Two individuals, 1 and 2, two alternatives, f and g, and two probable states of the world, s and t.

	s	t
f	$(1,0)$	$(1,0)$
g	$(1,0)$	$(0,1)$

Assuming that:

- society is SEU over $\{s,t\}$,
- social utility function is a uniform combination of individual utilities,

society is indifferent between f and g.

Yet Diamond writes:
"*However,* [g] *seems strictly preferable to me, since it gives* [individual 2] *a fair shake while* [f] *does not.*"

Diamond example in extended state space

Individual state space: $S = \{s, t\}$.

Social state space: $\Omega = \{(s,s),(s,t),(t,s),(t,t)\}$.

	(s,s)	(s,t)	(t,s)	(t,t)
f	$(1,0)$	$(1,0)$	(1,0)	(1,0)
g	$(1,0)$	$(1,1)$	(0,0)	(0,1)

Now, SEU maximization over Ω can be compatible with the choice of g if social probability of state (s,t) is high enough.

Diamond example: Conclusions

In the Diamond example, what makes the choice of g impossible is the implicit assumption of consequentialism.

Consequentialism says that any two acts that always induce indifferent outcomes should be indifferent *ex ante*. It forces society to evaluate a social prospect in terms of *actual outcomes*.

Resorting to the notion of social states introduces non-consequentialist motives: society evaluates a social prospect in terms of *anticipated outcomes*.

Father example in extended state space

Individual state space: $S = \{A, B\}$.

Social state space: $\Omega = \{(A, A), (A, B), (B, A), (B, B)\}$.

Social probability is the independent product of $(\frac{1}{3}, \frac{2}{3})$ and $(\frac{2}{3}, \frac{1}{3})$.

Social utility is the unweighted mean of individual utilities.

	(A,A)	(A,B)	(B,A)	(B,B)
λ_0	2/9	1/9	4/9	2/9
BA	(3, 3)	(3, 3)	(3,3)	(3,3)
PhD	(8, 0)	(8, 8)	(0,0)	(0,8)

This presentation

Framework – Individual preferences

A finite set N of individuals.

A set (S, Σ) of states of the world.

A set X of outcomes.

Let $F(S)$ denote the corresponding set of acts.

Each individual $i \in N$ has preferences $\succsim_i$ over $F(S)$.

Each $\succsim_i$ is assumed SEU with respect to utility u_i and probability λ_i.

Ex post preferences are denoted by $\succsim_i^E$ for $i \in N$ and $E \in \Sigma$.

Framework – Social preferences

Let $\Omega = S^N$ stand for the set of social states and $\mathcal{F}$ stand for the product σ-algebra.

Let X^N be the set of social outcomes.

Let $F(\Omega)$ denote the corresponding set of acts.

Society has preferences $\succsim_0$ over $F(\Omega)$.

Preferences $\succsim_0$ are assumed SEU with respect to utility u_0 and probability λ_0.

Ex post preferences are denoted by $\succsim_0^{\mathcal{E}}$ for $\mathcal{E} \in \mathcal{F}$.

Decision procedure

Society has preferences $\succsim_0$ over Ω but "real alternatives" are defined over S. Then, how should society use its preferences to make decisions?

Fix $f \in F(S)$ and define the social act $f \otimes ... \otimes f \in F(\Omega)$ as follows:

$$\forall \omega \in \Omega \text{ by } (f \otimes ... \otimes f)(\omega) = (f(\omega_1), ..., f(\omega_n))$$

For $f, g \in F(S)$, society weakly chooses f over g iff $f \otimes ... \otimes f \succsim_0 g \otimes ... \otimes g$.

Aggregation procedure

Aggregation procedure:

- For all $E_1, ..., E_N \in \Sigma$, $\lambda_0(E_1 \times ... \times E_N) = \lambda_1(E_1) \times ... \times \lambda_n(E_n)$
- For all $x_1, ..., x_N \in X$, $u_0(x_1, ..., x_N) = \alpha_1 u_1(x_1) + ... + \alpha_n u_n(x_n)$,

The social value of $f \in F(S)$ is given by:

$$\mathbb{E}_{\lambda_0} u_0(f \otimes ... \otimes f) = \sum_{i \in N} \alpha_i \mathbb{E}_{\lambda_i} u_i(f)$$

Axiomatic characterization

Let $\tilde{F}(\Omega)$ be the set of social acts of the form $f_1 \otimes ... \otimes f_N$.

The extended Pareto condition

Let $F, G \in F(\Omega)$ and $\tilde{F}, \tilde{G} \in \tilde{F}(\Omega)$. Let $(E_i)_{i \in N} \in \Sigma^N$ be such that $\lambda_0(\mathcal{E}) > 0$ where $\mathcal{E} = E_1 \times ... \times E_n$.

(1) If, for any $i \in N$ and $\omega \in \Omega$, $F_i(., \omega_{-i}) \succsim_i^{E_i} G_i(., \omega_{-i})$, then $F \succsim_0^{\mathcal{E}} G$.

(2) If, for any $i \in N$ and $\omega \in \Omega$, $\tilde{F}_i(., \omega_{-i}) \succsim_i^{E_i} \tilde{G}_i(., \omega_{-i})$ and there exists $i \in N$ such that, for any $\omega \in \Omega$, $\tilde{F}_i(., \omega_{-i}) \succ_i^{E_i} \tilde{G}_i(., \omega_{-i})$, then $\tilde{F} \succ_0^{\mathcal{E}} \tilde{G}$.

Theorem

The extended Pareto condition holds iff λ_0 is the independent product of $(\lambda_i)_{i \in N}$ and u_0 is a convex combination of $(u_i)_{i \in N}$ with positive coefficients.

Sketch of proof

Fix $(y_i)_{i \in N} \in X^N$ and $(E_i)_{i \in N} \in \Sigma^N$. Let $\mathcal{E} = E_1 \times ... \times E_N \in \mathcal{F}$.

Fix $i \in N$ and use the extended Pareto condition to show:

$$\forall f_i, g_i \in F(S),\ f_i \succsim_i^{E_i} g_i \Longleftrightarrow (f_i, y_{-i}) \succsim_0^{\mathcal{E}} (g_i, y_{-i})$$

Both sides of the the equivalence are SEU.

The uniqueness part of the SEU representation implies:

- for all $x \in X$, $u_0(x, y_{-i}) = \alpha_i(y_1, ..., y_n) u_i(x) + \beta_i(y_1, ..., y_n)$.
- $\lambda_i(.|E_i)$ is the i-th marginal of $\lambda_0(.|\mathcal{E})$